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Decentralized Adaptive Control of Large Scale Systems, with Application to Robotics

Donald T. Gavel, Jr.

(Ph.D. Thesis)

March 30, 1988

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Decentralized Adaptive Control of Large Scale Systems, with Application to Robotics

By

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Electrical Engineering
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Decentralized Adaptive Control of Large Scale Systems, with Application to Robotics

Abstract

Decentralized control has the advantages of simplicity and robustness over centralized control. By adding an adaptation mechanism to the decentralized law, further robustness to uncertain interconnections and subsystem dynamics is attained. In this thesis, strutural characteristics of the interconnection structure are exploited in order to prove the global stablity of decentralized adaptive control laws. Provided the structural constraints are satisfied, adaptation gains will automatically adjust to levels that assure stability of the overall system. Several example simulations are given to illustrate the operation of closed-loop controllers using this approach. To further demonstrate the viability of the approach, the adaptive state feedback algorithm was implemented as the joint servo controller on a PUMA robot arm at the Robotics Research Laboratory. Experimental results with this algorithm, along with comparison runs using a fixed decentralized controller, are presented.

To Nancy and Cynthia

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1
Introduction

Present day economic, technological, and environmental systems are large and complex. Gaining an understanding of large scale systems, that is, modeling their behavior and designing appropriate stabilizing controls, is a foremost challenge of modern system theory. One approach to large scale system modeling and control is decomposition of the large system into smaller, more manageable units. This is known as the decentralized approach. Decentralized control schemes have proven to be robust to a large range of uncertainties and nonlinearities in interconnections and subsystem dynamics. For the purpose of decentralized control, decompositions of large scale systems are typically formulated to isolate uncertainty about system behavior to the interaction between subsystems. Thereby the subsystems themselves are well modeled and decentralized controllers can be designed according to standard techniques.

It is often the case however that such an arrangement is impossible, and that subsystems parameters are only poorly known, or not known at all. For this reason, adaptive decentralized control schemes have been of recent interest. Adaptive control algorithms offer the benefit of being robust to uncertain subsystem dynamics, whereas the large scale methodology offers simpler control designs and robustness to interconnection strengths. Additional robustness to interconnection strengths is afforded through the adaptation mechanism, which tends to increase local subsytem stability such that the overall system is stabilized regardless of the strength of interconnections.

In this thesis, we develop the theory of decentralized adaptive control for decentrally stabilizable systems. The new schemes depend upon local high gain feedback to stabilize local systems sufficiently to overcome interconnection disturbances, leading to a stable overall system.

The question of decentralized stabilizability was first studied by Wang and Davison where it was tied to the presence or absence of fixed modes [1]. Siljak [2] introduced a robust form of stabilizability called structural stabilizability, and the corresponding structurally fixed modes. Tests for structural stabilizability were developed, along with constructive methods for the control design [3-7].

The design stage usually requires parametric information about the subsystems and interconnections in order for feedback gains to be properly chosen. In this thesis, we assume that such information is unavailable, and resort to adaptive methods for automatically chosing the gains. It turns out that in all cases where a fixed decentralized control design is possible because of decentralized stabilizability, an adaptive controller will also work. The required a-priori knowledge is limited to upper bounds on the norms of the system and interconnection matrices. Furthermore, in a certain subset of cases, even these norm bounds are not necessary to guarantee a stable design.

We use the model reference adaptive control (MRAC) approach in our study. In this approach, feedback gains are tuned during plant operation with the objective of matching closed loop system response to that of a reference model. A survey of model reference adaptive control is given in [8]. Parks [9] first introduce a systematic design procedure for adaptive controllers based on the second method of Liapunov [10]. Monopoli [11] later extended Parks idea to a general class of plants.

Further work lead to solid stability proofs [12], discrete time versions [13], relaxed conditions on the plant [14,15], conditions for convergence of the parameter estimates [16,17], allowance for unmodeled high frequency dynamics [18], and extension to multi-input multi-output systems [19].

A decentralized adaptive regulator was first proposed by Davison [20]. Later, as the adaptive control theory developed more decentralized adaptive schemes were developed [21,22]. The structural stabilizability conditions for decentralized adaptive control were first reported in [23,24].

The success of both decentralized and adaptive control in applications has been well established. Decentralized controllers have been applied to the control of electric power networks [25] and spin stabilization of the Skylab spacecraft [25]. Adaptive controllers have been implemented in several process control applications, including the control of a paper mill [26], and an ore crusher [27].

Recently, there has been much interest in decentralized adaptive control of robot manipulators. Experimental work using an industrial robot arm has been done by Seraji [28] and Gavel and Hsia [29].

The thesis is orgainized as follows. We first introduce the decentralized adaptive control problem in a formal sense in Chapter 2, and then develop an adaptive controller along with a proof of stability for a restriced class of decentrally stabilizable systems in Chapter 3. In Chapter 4 we recognize a spinoff benefit for the case where subsystem dynamics are known to the designer, but where bounds on interconnection strengths (required in the fixed control design) are unknown, and develop a suitable adaptive controller. In Chapter 5 the results of Chapter 3 are extended to the case where only output variables are available for feedback, and dynamic compensators must be used in the control law. In Chapter 6 we again return to the state feedback case but now consider the wider class of decentrally stabilizable systems. In Chapter 7 we apply the theory to robot manipulators, and in Chapter 8 present the results of laboratory experiments with a six degree of freedom robot arm. Appendices A and B are provided as tutorials on the large scale system and adaptive control theories respectively.

2
Problem Statement

In this chapter, we formally state the decentralized adaptive control problem. We give a mathematical description of a large scale system in terms of its component subsystems and subsystem interconnections. This will lead naturally to the adaptive control algorithms and proofs of stability given in subsequent chapters.

Consider a collection of N subsystems

$$\begin{aligned} \mathbf{S}_i : \dot{x}_i &= A_i x_i + b_i u_i + P_i v_i \\ y_i &= c_i^T x_i \qquad\qquad i \in \boldsymbol{N} \\ w_i &= Q_i x_i \end{aligned} \tag{2.1}$$

where $x_i(t) \in \mathcal{R}^{n_i}$, $u_i(t) \in \mathcal{R}$ and $y_i(t) \in \mathcal{R}$ are the state, input, and output of $\mathbf{S}_i$ at time $t \in \mathcal{R}$, and $\boldsymbol{N} = \{1, 2, \ldots, N\}$. In (2.1), $v_i \in \mathcal{R}^{m_i}$ and $w_i \in \mathcal{R}^{l_i}$ are the interconnection inputs and outputs of $\mathbf{S}_i$ from and to other subsystems $\mathbf{S}_j$, $j \in \boldsymbol{N}$, at time $t \in \mathcal{R}$, which are related as

$$v_i = f_i(t, w), \quad i \in \boldsymbol{N} \tag{2.2}$$

where $w = (w_1^T, w_2^T, \ldots, w_N^T)^T$.

The overall system $\mathbf{S}$, which is composed of subsystems $\mathbf{S}_i$ interconnected as in (2.2), can be given in a compact form

$$\begin{aligned} \mathbf{S} : \dot{x} &= A_D x + Bu + Pv \\ y &= Cx \\ w &= Qx \end{aligned} \tag{2.3}$$

where $x(t) \in \mathcal{R}^n$, $u(t) \in \mathcal{R}^N$, and $y(t) \in \mathcal{R}^N$ are the state $x = (x_1^T, x_2^T, \ldots, x_N^T)^T$, input $u = (u_1^T, u_2^T, \ldots, u_N^T)^T$, and output $y = (y_1^T, y_2^T, \ldots, y_N^T)^T$ of $\mathbf{S}$ at time $t \in \mathcal{R}$.

The constant block diagonal matrices

$$
\begin{aligned}
A_D &= \mathrm{diag}\{A_1, A_2, \ldots, A_N\} \\
B &= \mathrm{diag}\{b_1, b_2, \ldots, b_N\} \\
C &= \mathrm{diag}\{c_1^T, c_2^T, \ldots, c_N^T\} \\
P &= \mathrm{diag}\{P_1, P_2, \ldots, P_N\} \\
Q &= \mathrm{diag}\{Q_1, Q_2, \ldots, Q_N\}
\end{aligned}
\tag{2.4}
$$

have appropriate dimensions. The vectors $v(t) \in \mathcal{R}^m$ and $w(t) \in \mathcal{R}^l$, which are defined as $v = (v_1^T, v_2^T, \ldots, v_N^T)^T$, and $w = (w_1^T\ w_2^T, \ldots, w_N^T)^T$ are interconnected as

$$v = f(t, w) \tag{2.5}$$

The function $f : \mathcal{R} \times \mathcal{R}^m \rightarrow \mathcal{R}^l$ is continuous and bounded in both arguments, and sufficiently smooth so that the solution $x(t; t_0, x_0)$ of $\mathbf{S}$ is unique for all initial conditions $(t_0, x_0) \in \mathcal{R} \times \mathcal{R}^n$ and all piece-wise continuous inputs $u(\cdot)$. In particular, we assume that there exist nonnegative numbers ξ_{ij} such that

$$\|f_i(t, w)\| \leq \sum_{j=1}^{N} \xi_{ij}\, \|w_j\| \quad \forall (t, w) \in \mathcal{R} \times \mathcal{R}^m,\ i \in \boldsymbol{N}. \tag{2.6}$$

where $\|\cdot\|$ indicates the Euclidean norm of the indicated vector. Henceforth $\|\cdot\|$ will denote the Euclidean norm if the argument is a vector, and the Spectral norm if the argument is a matrix.

We presume that neither the dynamics of the subsystems, nor the strength of the interconnections are known *a-priori*. That is, the elements of the matrices A_i, b_i, and c_i, and the numbers ξ_{ij}, are not specified to the designer beforehand. We can only assume that the pairs (A_i, b_i) are controllable and the pairs (A_i, c_i) are observable (which is equivalent to assuming the order of each subsystem). The controllability assumption guarantees a choice of the coordinate system such that the matrices of the decoupled systems have the companion form

$$A_i = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ -a_1^i & -a_2^i & -a_3^i & \dots & -a_{n_i}^i \end{pmatrix}, b_i = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ b^i \end{pmatrix} \tag{2.7}$$
$$c_i^T = (\quad c_1^i \quad c_2^i \quad \dots \quad c_{n_i}^i) .$$

The uncertainty about the subsystems $\mathbf{S}_i$, which is reduced to the unspecified coefficients of A_i, b_i, and c_i, compounds an essential uncertainty about the overall system $\mathbf{S}$ caused by our ignorance of the interconnection strengths ξ_{ij}.

A way to overcome the uncertainty about the interconnection strengths in $\mathbf{S}$ is to exploit any special structural features of $\mathbf{S}$ which make it a decentrally stabilizable system [4], that is, a system where there exists a *fixed* stabilizing decentralized control law for any *fixed* interconnection strength. We shall show in subsequent chapters that such a structural characteristic ensures the existence of adaptive decentralized laws which can stabilize $\mathbf{S}$ by adjusting the local gains to compensate for unknown, *fluctuating* levels of the interconnections as well as unknown subsystem dynamics.

The control objective is to either regulate the state, $x(t)$ of the systems $\mathbf{S}$ to zero, or force $x(t)$ to track the state of a given reference model. A model for $\mathbf{S}$ is formed by assigning to each subsystem $\mathbf{S}_i$ a local model

$$\begin{aligned} \mathbf{M}_i : \dot{x}_{mi} &= A_{mi} x_{mi} + b_{mi} r_i \\ y_{mi} &= c_{mi}^T x_{mi} \end{aligned} \qquad i \in N \tag{2.8}$$

having the same dimensions as $\mathbf{S}_i$, and stability characteristics we would like $\mathbf{S}_i$ to have. By stacking up the individual models $\mathbf{M}_i$, we obtain a model $\mathbf{M}$ for the overall system $\mathbf{S}$,

$$\begin{aligned} \mathbf{M} : \dot{x}_m &= A_m x_m + b_m r \\ y_m &= c_m^T x_m , \end{aligned} \tag{2.9}$$

where $r(t) \in \mathcal{R}^N$ is the external (reference) input of $\mathbf{M}$. As in (2.4), we have

$$
\begin{aligned}
A_m &= \mathrm{diag}\{A_{m1}, A_{m2}, \ldots, A_{mN}\} \\
B_m &= \mathrm{diag}\{b_{m1}, b_{m2}, \ldots, b_{mN}\} \\
C_m &= \mathrm{diag}\{c_{m1}^T, c_{m2}^T, \ldots, c_{mN}^T\}
\end{aligned}
\tag{2.10}
$$

Coordinates for each local model $\mathbf{M}_i$ are chosen so that the triples $(A_{mi}, b_{mi}, c_{mi}^T)$ are in the companion (controller) canonical form as in (2.7). With this choice of coordinates, it is clear that there exist constant vectors $k_i^* \in \mathcal{R}^{n_i}$, $k_{0i}^* \in \mathcal{R}$, such that

$$
A_{mi} = A_i + b_i k_i^{*T}, \quad b_{mi} = b_i k_{0i}^*. \tag{2.11}
$$

These model-matching parameter settings are, however, unknown due to the fact that the subsystem dynamics are unknown. We only assume that we know the sign of b^i and, without loss of generality, can set $k_{0i}^* > 0$ for all $i \in N$. It is important to note that, because of the interconnections among the subsystems, the ideal setting of parameters may not be such that the isolated subsystems match the corresponding models. Instead, different, possibly higher gain settings may have to be chosen by the adaptation mechanism in order to cancel the interconnection disturbances.

3

State Feedback Adaptive Control

We are now in a position to define an interesting class of decentrally stabilizable systems, and proceed to derive a state feedback decentralized adaptive controller for that system. This will be followed by a proof of global stability.

Assume that the interconnection signals enter the subsystems only through the range space of the control variables. This condition is satisfied if every matrix P_i has b_i as a left factor, that is

$$P_i = b_i p_i, \quad i \in N \tag{3.1}$$

for some constant vectors, $p_i \in \mathcal{R}^{m_i}$. This special structure for interconnections is a sufficient, but not necessary, condition for the system to be stabilizable *via fixed* decentralized control [7]. We will exploit it here and in the next two chapters in proving global stability of proposed *adaptive* control laws.

We start with the *state regulation problem* where it is desired to maintain the state of system at a constant setting. Without loss of generality, we can assume that the desired equilibrium state of the overall system $\mathbf{S}$ is zero. To drive the system state always toward zero, we use the local state feedback control laws

$$u_i = \theta_i^T x_i, \quad i \in N \tag{3.2}$$

where $\theta_i(t) \in \mathcal{R}^{n_i}$ is a time-variable adaptation gain vector at time $t > t_0$.

To design a suitable adaptation law for $\theta_i(t)$, we choose a stable reference model $\mathbf{M}_i$ whose dynamic behavior we would like to have matched by the closed-loop system. We then choose any symmetric positive definite matrix G_i and solve the Liapunov matrix equation

$$A_{mi}^T H_i + H_i A_{mi} = -G_i, \quad i \in N \tag{3.3}$$

for H_i. Equation (3.3) is quaranteed to have a unique symmetric postive definite solution for H_i since A_{mi}, which is associated with the reference model, is a stable matrix [30]. Software is commonly available for solving (3.3) numerically [31].

Next, we define $\bar{k}_i \in \mathcal{R}^{n_i}$ as

$$\bar{k}_i = H_i b_{mi}, \quad i \in N \tag{3.4}$$

and propose to use the local adaptation laws

$$\dot{\theta}_i = -\Gamma_i(\bar{k}_i^T x_i)x_i, \quad i \in N \tag{3.5}$$

where Γ_i is an $n_i \times n_i$ constant symmetric positive definite matrix, and $\theta_i(t_0)$ is finite.

To summarize, the closed-loop system is described as

$$\hat{\mathbf{S}} : \begin{array}{l} \dot{x}_i = (A_{mi} + b_i\phi_i^T)x_i + b_i p_i^T f_i \\ \dot{\phi}_i = -\Gamma_i(\bar{k}_i^T x_i)x_i \end{array} \quad i \in N \tag{3.6}$$

where $\phi_i(t) \in \mathcal{R}^{n_i}$ is the ith parameter adaptation error at time $t \in \mathcal{R}$, defined as

$$\phi_i = \theta_i - \theta_i^*, \quad i \in N \tag{3.7}$$

and $\theta_i^* = k_i^*$ is the constant, but unknown, model matching gain vector that satisfies equation (2.11). We define $\phi = (\phi_1^T, \phi_2^T, \ldots, \phi_N^T)^T$ and denote the state of $\hat{\mathbf{S}}$ as $(x, \phi) \in \mathcal{R}^n \times \mathcal{R}^n$.

(3.8) **THEOREM.** The solutions $(x, \phi)(t; t_0, x_0, \phi_0)$ of the system $\hat{\mathbf{S}}$ are globally bounded, and $x(t; t_0, x_0, \phi_0) \to 0$ as $t \to \infty$.

PROOF. We define a function $V : \mathcal{R}^n \times \mathcal{R}^n \to \mathcal{R}_+$ as

$$V(x, \phi) = \sum_{i=1}^{N} \left[k_{0i}^* x_i^T H_i x_i + (\phi_i + \rho\bar{k}_i)^T \Gamma_i^{-1}(\phi_i + \rho\bar{k}_i)\right], \tag{3.9}$$

where the number $\rho > 0$ is not yet specified. The total time derivative $\dot{V}(x, \phi)_{(3.6)}$ of the function $V(x, \phi)$ which is computed with respect to (3.6), is obtained as

$$\dot{V}(x, \phi)_{(3.6)} = -x^T G x - 2\rho x^T \bar{K}^T \bar{K} x + 2x^T \bar{K}^T \tilde{P} f \tag{3.10}$$

where

$$
\begin{aligned}
G &= \mathrm{diag}\{k_{01}^* G_1, k_{02}^* G_2, \ldots k_{0N}^* G_N\} \\
\tilde{P} &= \mathrm{diag}\{p_1^T, p_2^T, \ldots, p_N^T\} \\
\bar{K} &= \mathrm{diag}\{\bar{k}_1^T, \bar{k}_2^T, \ldots, \bar{k}_N^T\}.
\end{aligned}
\tag{3.11}
$$

Completing the square in (3.10) we get

$$
\begin{aligned}
\dot{V}_{(3.6)}(x, \phi) = &- x^T G x - \rho x^T \bar{K}^T \bar{K} x \\
&- \rho(\bar{K}x - \rho^{-1}\tilde{P}f)^T(\bar{K}x - \rho^{-1}\tilde{P}f) + \rho^{-1} f^T \tilde{P}^T \tilde{P} f.
\end{aligned}
\tag{3.12}
$$

By using the boundedness assumption (2.6) concerning $f(t, w)$, we get from (3.11) the inequality

$$
\begin{aligned}
\dot{V}(x, \phi)_{(3.6)} \leq - \left[\lambda_{\min}(G) - \rho^{-1}\xi^2 \|\tilde{P}\|^2 \|Q\|^2\right] \|x\|^2 \\
\forall (t, x, \phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n
\end{aligned}
\tag{3.13}
$$

where $\lambda_{\min}(G)$ is the minimum eigenvalue of G, and $\xi = \max_{i,j \in N} \xi_{ij}$. By selecting a sufficiently large finite value ρ^* of ρ so that

$$
\rho^* > \lambda_{\min}^{-1}(G)\xi^2 \|\tilde{P}\|^2 \|Q\|^2,
\tag{3.14}
$$

we get $\dot{V}(x, \phi)_{(3.6)} \leq 0$ for all $(t, x, \phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n$. Using standard arguments from the Liapunov's theory (*e.g.*, [10]) we conclude that the solutions $(x, \phi)(t; t_0, x_0, \phi_0)$ are bounded.

From the boundedness of solutions and definition (3.9) of $V(x, \phi)$, we conclude that $\dot{V}(x, \phi)_{(3.6)}$ is uniformly continuous on $\mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n$. Furthermore, the function $V(t) = V[x(t), \phi(t)]$ is decreasing and is bounded from below. Hence,

$$
\lim_{t \to \infty} V(t) = \inf_t V(t) = V_f \geq 0.
\tag{3.15}
$$

Denoting $\dot{V}(t) = \dot{V}[x(t), \phi(t)]_{(3.6)}$, we have

$$
\lim_{t \to \infty} \int_{t_0}^{t} \dot{V}(\tau) d\tau = V_f - V_0 < \infty
\tag{3.16}
$$

where $V_0 = V(x_0)$. Then, from (3.9), (3.10), and uniform continuity of $\dot{V}(t)$, we have $\lim_{t\to\infty} \dot{V}(t) = 0$, and $\lim_{t\to\infty} x(t; t_0, x_0, \phi_0) = 0$ for all $(t_0, x_0, \phi_0) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n$. **Q.E.D.**

(3.17) **REMARK.** It is not necessary to know or choose ρ^* in (3.9). The gains of the controllers rise to whatever level is necessary to ensure that stability of the subsystems overrides the perturbations caused by interconnections regardless of their size so long they are finite. This is a pleasing result because the essential uncertainty in large composite systems not only resides in the interconnections, but their size and characterization is neither known nor can be predicted during the operation of the overall system [25]

(3.18) **REMARK.** The boundedness part of Theorem (3.8) assures the boundedness of the adaptation gains and, thus, realizability of the decentralized adaptive control scheme.

In the *state tracking* problem, it is required that the state $x(t)$ of the plant **S** follows the state of the reference model **M** despite the fluctuating interaction levels among the subsystems $\mathbf{S}_i$. Let the tracking error be defined as

$$e(t) = x(t) - x_m(t), \tag{3.19}$$

where $e = (e_1^T, e_2^T, \ldots, e_N^T)^T$ and $e_i(t) \in \mathcal{R}^{n_i}$ is the tracking error of the subsystem $\mathbf{S}_i$ at $t \in \mathcal{R}$. To drive the error toward zero, we use the local control laws

$$u_i = \theta_i^T \nu_i, \tag{3.20}$$

where $\theta_i \in \mathcal{R}^{n_i+1}$:

$$\theta_i = (\hat{k}_i^T, \hat{k}_{0i})^T \tag{3.21}$$

and $\nu_i \in \mathcal{R}^{n_i+1}$:

$$\nu_i = (e_i^T, r_i^T)^T. \tag{3.22}$$

The gains $\hat{k}_i$ and $\hat{k}_{0i}$ represent the estimates of k_i^* and k_{0i}^*.

(3.23)**REMARK.** This is a deviation from the standard choice of state regressor $\nu_i = (x_i^T, r_i^T)^T$ used in centralized [32] and decentralized [33] adaptive control. Instead of the state $x_i(t)$ we choose to feed back the tracking error $e_i(t)$. This fact is crucial for the functioning of our scheme.

The resulting closed-loop system is

$$\hat{\mathbf{S}} : \dot{x}_i = A_{mi}x_i + b_{mi}r_i + b_i(\phi_i^T\nu_i - k_i^{*T}x_{m_i} + p_i^T f_i), \quad i \in N \tag{3.24}$$

where we have defined $\phi_i(t)$ as in (3.7), and

$$\theta_i^* = (k_i^{*T}, k_{0i}^*)^T. \tag{3.25}$$

Subtracting (2.7) from (3.24), we can write the error system as

$$\hat{\mathbf{S}}_e : \begin{aligned} \dot{e}_i &= A_{mi}e_i + b_i(\phi_i^T\nu_i - k_i^{*T}x_{mi} + p_i^T f_i) \\ \dot{\phi}_i &= -\sigma\Gamma_i\phi_i - \Gamma_i(\bar{k}_i^T e_i)\nu_i - \sigma\Gamma_i\theta_i^* \end{aligned} \quad i \in N \tag{3.26}$$

where $\sigma > 0$. The adaptation law was chosen as

$$\dot{\theta}_i = -\Gamma_i(\bar{k}_i^T e_i)\nu_i - \sigma\Gamma_i\theta_i, \quad i \in N \tag{3.27}$$

which incorporates the "σ-modification", as originally suggested in [18]. By $(e,\phi) \in \mathcal{R}^n \times \mathcal{R}^{n+N}$ we donote the state of $\hat{\mathbf{S}}_e$ and prove:

(3.28) **THEOREM.** The solutions $(e,\phi)(t; t_0, e_0, \phi_0)$ of the system $\hat{\mathbf{S}}_e$ are globally ultimately bounded.

PROOF. We use the same type of function $V : \mathcal{R}^n \times \mathcal{R}^{n+N} \to \mathcal{R}_+$ as in (3.9).

$$V(e,\phi) = \sum_{i=1}^{N}\left[k_{0i}^* e_i^T H_i e_i + (\phi_i + \rho\bar{\theta}_i)^T\,\Gamma_i^{-1}\,(\phi_i + \rho\bar{\theta}_i)\right], \tag{3.29}$$

where $\bar{\theta}_i = (\bar{k}_i^T, 0)^T \in \mathcal{R}^{n_i+1}$, and ρ is a positive number. We compute

$$\begin{aligned} \dot{V}(e,\phi)_{(3.26)} = \sum_{i=1}^{N}\Big[&- k_{0i}^* e_i^T G_i e_i - 2e_i^T\bar{k}_i k_i^{*T}x_{mi} + 2e_i^T\bar{k}_i p_i^T f_i \\ &- 2\rho e_i^T\bar{k}_i\bar{k}_i^T e_i - 2\sigma(\phi_i + \rho\bar{\theta}_i)^T\theta_i\Big] \end{aligned} \tag{3.30}$$

where we have used the fact that $\bar{\theta}_i^T \nu_i = \bar{k}_i^T e_i$ Utilizing the block notation of (3.11), we can rewrite (3.30) as

$$\begin{aligned} \dot{V}(e,\phi)_{(3.26)} = &- e^T G e - 2e^T \bar{K}^T K^* x_m + 2e^T \bar{K}^T \tilde{P} f - 2\rho e^T \bar{K}^T \bar{K} e \\ &- 2\sigma(\phi + \rho\bar{\theta})^T(\phi + \rho\bar{\theta}) - 2\sigma(\phi + \rho\bar{\theta})^T(\theta^* - \rho\bar{\theta}) \end{aligned} \tag{3.31}$$

where $K^* = \text{diag}\left\{k_1^{*T}, k_2^{*T}, \ldots, k_N^{*T}\right\}$. Since $f(t, w) = f\left[t, Q(e + x_m)\right]$ is bounded as in (2.6), we can use

$$\|f_i(t,w)\| \leq \sum_{j=1}^{N} \xi_{ij} \|Q_j\| \left(\|e_i\| + \|x_{mi}\|\right) \tag{3.32}$$

in (3.31) to get the inequality

$$\begin{aligned} \dot{V}(e,\phi)_{(3.6)} \leq &- e^T G e - 2\rho \left\|\bar{K} e\right\|^2 + 2 \left\|\bar{K} e\right\| \|K^* x_m\| \\ &+ 2\xi \left\|\bar{K} e\right\| \left\|\tilde{P}\right\| \|Q\| \left(\|e\| + \|x_m\|\right) - 2\sigma \left\|\phi + \rho\bar{\theta}\right\|^2 \\ &+ 2\sigma \left\|\phi + \rho\bar{\theta}\right\| \left\|\theta^* - \rho\bar{\theta}\right\| \quad \forall (t, e, \phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n. \end{aligned} \tag{3.33}$$

After completing the squares involving $\bar{K} e$ and $\phi + \rho\bar{\theta}$, and dropping negative terms, we obtain

$$\begin{aligned} \dot{V}(e,\phi)_{(3.6)} \leq &- \left[\lambda_{\min}(G) - \rho^{-1}\xi^2 \left\|\tilde{P}\right\|^2 \|Q\|^2\right] \|e\|^2 - \sigma \left\|\phi + \rho\bar{\theta}\right\|^2 \\ &+ \rho^{-1}\chi \left(\|K^*\| + \xi \left\|\tilde{P}\right\| \|Q\|\right)^2 + \sigma \left\|\theta^* - \rho\bar{\theta}\right\|^2 \\ &\forall (t, e, \phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n \end{aligned} \tag{3.34}$$

where $\chi = \sup_t \|x_m(t)\|^2$.

Equation (3.34) can be written compactly as

$$\begin{aligned} \dot{V}(e,\phi)_{(3.6)} \leq -\zeta \|e\|^2 - \sigma \left\|\phi + \rho\bar{\theta}\right\|^2 + \eta \\ \forall (t, e, \phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n, \end{aligned} \tag{3.35}$$

where the constants ζ and η are defined

$$\begin{aligned} \zeta &= \lambda_{\min}(G) - \rho^{-1}\xi^2 \left\|\tilde{P}\right\|^2 \|Q\|^2 \\ \eta &= \rho^{-1}\chi \left(\|K^*\| + \xi \left\|\tilde{P}\right\| \|Q\|\right)^2 + \sigma \left\|\theta^* - \rho\bar{\theta}\right\|^2. \end{aligned} \tag{3.36}$$

Selecting ρ^* large enough so that $\zeta > 0$, we see that (3.35) implies

$$\dot{V}(e,\phi)_{(3.6)} \leq -\mu V(e,\phi) + \eta \quad \forall (t,e,\phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n, \tag{3.37}$$

where the positive number μ is given by

$$\mu \leq \min\left[\lambda_{\max}^{-1}(H)\zeta,\ \lambda_{\min}(\Gamma)\sigma\right], \tag{3.38}$$

where $H = \operatorname{diag}\{k_{o1}^* H_1, k_{o2}^* H_2, \ldots, k_{oN}^* H_N\}$ and $\Gamma = \operatorname{diag}\{\Gamma_1, \Gamma_2, \ldots, \Gamma_N\}$. From (3.37) it is clear that $V(e,\phi)$ decreases monotonically along any solution of $\hat{\mathbf{S}}_e$ until the solution reaches the compact set

$$\Omega_f = \left\{(e,\phi) \in \mathcal{R}^n \times \mathcal{R}^{n+N} : V(e,\phi) \leq V_f\right\} \tag{3.39}$$

where

$$V_f = \mu^{-1}\eta. \tag{3.40}$$

Therefore, the solutions $(e,\phi)(t;t_0,e_0,\phi_0)$ of $\hat{\mathbf{S}}_e$ are globally ultimately bounded with respect to the bound V_f. **Q.E.D.**

(3.41) **REMARK.** Within Ω_f we find that

$$\|e\| \leq \left[\lambda_{\min}^{-1}(H)V_f\right]^{1/2} \tag{3.42}$$

Now, if we choose $\sigma \propto \rho^{-3}$ and $\Gamma \propto \rho^3$, and let $\rho \to +\infty$, we find that $\zeta \to \lambda_{\min}(G)$, $\eta \sim \rho^{-1}$, and, therefore, $V_f \sim \rho^{-1}$. This implies that the upper bound on the steady-state tracking error $e(t)$ can be made as small as desired by decreasing σ and increasing Γ sufficiently. We should note, however, that within Ω_f,

$$\left\|\phi + \rho\bar{\theta}\right\| \leq \left[\lambda_{\max}(\Gamma)V_f\right]^{1/2} \tag{3.43}$$

and the upper bound on $\left\|\phi + \rho\bar{\theta}\right\|$ increases proportionally to the increase of ρ. Thus, making σ small and Γ large will allow adaptation gains to become high, resulting in a trade-off between small tracking errors and large gains in the proposed high-gain decentralized adaptive scheme.

We also note that again, as in Remark (3.17), it is not necessary for the designer to know or choose ρ^*. Local gains adjust automatically to counter the destabilizing effect of interconnections. The designer need only to tune the size of the residual set by adjusting Γ_i and σ as explained in Remark (3.41).

(3.44) **EXAMPLE.** To illustrate the application of the proposed adaptive scheme, let us consider the two identical penduli which are coupled by a spring and subject to two distinct inputs as shown in Figure 3.1. The important feature of the system is that the position a of the spring can change along the full length l of the penduli. We want to design a decentralized control applied to the individual masses m, which can adapt to the changes in the spring position a.

Choosing the state vectors $x_1 = (\theta_1, \dot{\theta}_1)^T$ and $x_2 = (\theta_2, \dot{\theta}_2)^T$, the motion of the penduli can be described as

$$\begin{aligned} \mathbf{S}: \dot{x}_1 &= \begin{pmatrix} 0 & 1 \\ \frac{g}{l} & 0 \end{pmatrix} x_1 + \begin{pmatrix} 0 \\ \frac{1}{ml^2} \end{pmatrix} u_1 + \begin{pmatrix} 0 & 0 \\ -\frac{ka^2}{ml^2} & 0 \end{pmatrix} x_1 + \begin{pmatrix} 0 & 0 \\ \frac{ka^2}{ml^2} & 0 \end{pmatrix} x_2 \\ \dot{x}_2 &= \begin{pmatrix} 0 & 1 \\ \frac{g}{l} & 0 \end{pmatrix} x_2 + \begin{pmatrix} 0 \\ \frac{1}{ml^2} \end{pmatrix} u_2 + \begin{pmatrix} 0 & 0 \\ \frac{ka^2}{ml^2} & 0 \end{pmatrix} x_1 + \begin{pmatrix} 0 & 0 \\ -\frac{ka^2}{ml^2} & 0 \end{pmatrix} x_2 \end{aligned} \tag{3.45}$$

where k and g are spring and gravity constants. For our numerical example, we set $g/l = 1$, $1/ml^2 = 1$, and $k/m = 2$ Uncertainty in the interconnections is represented by making $a(t)$ an unknown function of time such that $a(t)/l \in [0, 1]$. Comparing (3.45) with (2.1), we get the subsystem matrices as

$$A_i = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; b_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \; P_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \; Q_i = \begin{pmatrix} 1 & 0 \end{pmatrix}, \; i = 1, 2. \tag{3.46}$$

In the block notation of (2.3), the system matrices are

$$A_D = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \; B_D = P_D = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \; Q_D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{3.47}$$

and the interconnection function is

$$f(t, w) = 2\alpha(t) \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} w, \tag{3.48}$$

where $\alpha(t) = a^2(t)/l^2$ and $w = (w_1, w_2)^T$

Since $P_i = b_i, i = 1,2$, the condition (3.1) for stabilizability of **S** by fixed decentralized feedback is satisfied. To build the adaptive controllers, we choose the reference models (2.8) having the matrices

$$A_{mi} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}, b_{mi} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, i = 1,2 \tag{3.49}$$

and $\theta_i^* = (-2,-2,1)^T, i = 1,2$. Next, we choose G_i and compute H_i and $\bar{k}_i$ from (3.3) and (3.4) as

$$G_i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, H_i = \begin{pmatrix} 1.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}, \bar{k}_i = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, i = 1,2 \tag{3.50}$$

For the adaptation law (3.5), we select $\Gamma = I_3$, where I_3 is the 3×3 identity matrix, and set $\sigma = 0.01$.

With the chosen parameter settings, the results of a simulation are shown in Figure 3.2. The reference signals were

$$\begin{aligned} r_1(t) &= \sin 20t + \sin 5t + \sin t \\ r_2(t) &= \sin 10t + \sin 2t + \sin 0.5t \end{aligned} \tag{3.51}$$

and $a(t)/l = 1$ for all time $t \geq t_0 = 0$. From the plots we see that the presence of interconnections prevents the tracking errors from converging to zero and the feedback gains from converging to the model matching values. Instead, the gains tend toward a near steady-state level, $\hat{k}_1 = (-3.9, -2.3)^T$, which is higher than the local model-matching gains of $k_1^* = (-2, -2)^T$, which is indicative of the high-gain nature of the controller.

To contrast the result of Theorem (3.28) with the earlier approach [22] based upon the $\mathcal{M}$-matrix conditions, we compute the 2×2 aggragate matrix $W = [w_{ij}]$ as

$$w_{ij} = \begin{cases} \lambda_{\min}(G_i) - 2\lambda_{\max}(H_i)\,\|P_i\|\,\xi_{ii}, & i = j \\ -2\lambda_{\max}(H_i)\,\|P_i\|\,\xi_{ij}. & i \neq j. \end{cases} \tag{3.52}$$

From the $\mathcal{M}$-matrix conditions [25] (see also appendix A) applied on W, we derive the inequality

$$\frac{\lambda_{\min}(G_i)}{\lambda_{\max}(H_i)} > 4\,\|P_i\|\,\xi_{ij}, \quad i = 1,2. \tag{3.53}$$

Using the definition of ξ_{ij} in (2.6), we have $\xi_{ij} = 2$ for $i, j = 1, 2$. Then, from (3.53) we get

$$\frac{a(t)}{l} < \frac{1}{2}\left[\frac{m\lambda_{\min}(G_i)}{k\,\|P_i\|\,\lambda_{\max}(H_i)}\right]^{1/2} = 0.2706, \tag{3.54}$$

which means that by $\mathcal{M}$-matrix approach we guarantee stability only if the spring remains connected to the lower 27% of the length of the penduli. In our example, we allow $\frac{a(t)}{l} \equiv 1$ (spring is moved all the way up to the bobs) making the $\mathcal{M}$-matrix test inconclusive with regard to stability

A simulation of the system using a controller design based on the $\mathcal{M}$-matrix test was also performed in order to see the effect of regressor vectors $\nu_i = (x_i^T, r_i^T)^T$ which are different from those of (3.22), $\nu_i = (e_i^T, r_i)^T$, used in our example. The results are shown in Figure 3.3. The system appears stable, however, tracking residuals are somewhat larger than before, indicating the fact that the high-gain approach may lead to a better performing adaptive control scheme.

In another simulation run, we show the effect of a time-varying interconnection on adaptation gains and tracking residuals. We inject the jump

$$\frac{a(t)}{l} = \begin{cases} 0, & 0 \le t < 50 \\ 1, & 50 \le t \le 100 \end{cases} \tag{3.55}$$

that is, the penduli are disconnected for the first half, and maximally interconnected for the second half of the experiment. Figure 3.4 shows the results for the high gain adaptive controllers, and Figure 3.5 shows the results for the standard adaptive scheme. The high-gain controllers maintain a tracking residual of about 0.5 throughout the run. As expected, the standard scheme behaves well when the subsystems are isolated. However, the residuals become large when the subsystems get coupled. The high-gain adaptive controllers seem robust to time-varying interconnections, with local gains adjusting their values to maintain a consistent tracking residual.

Finally, we compare the proposed decentralized scheme to the standard centralized adaptive algorithm [32]. For this purpose, we use the compact notation of the form (2.3) to describe the closed-loop system with $\frac{a(t)}{l} \equiv 1$ as

$$\hat{\mathbf{S}} : \dot{x} = Ax + Bu$$
$$u = Kx + K_0 r \tag{3.56}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & -1 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{3.57}$$

and K and K_0 are the gain matrices of appropriate dimensions. The model-matching gains are

$$K^* = \begin{pmatrix} 0 & -2 & -2 & 0 \\ -2 & 0 & 0 & -2 \end{pmatrix}, K_0^* = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.58}$$

Defining $\theta = (K, K_0)$ and $\nu = (x^T, r^T)^T$, the adaptation law is

$$\dot{\theta} = -\sigma\theta - \gamma \bar{K} e \nu \tag{3.59}$$

where $\bar{K} = \text{diag}\{\bar{k}_1^T, \bar{k}_2^T\}$ as defined by (3.4), $e = (e_1^T, e_2^T)^T$, and σ and γ are positive numbers. For simulation purposes, we chose $\gamma = 1$ which corresponds to $\Gamma = I_3$ in our earlier runs. We set $\sigma = 0$ because there are no external disturbances or unmodeled interconnections, so that perfect model-matching is possible. Our reference signal is persistently exciting, so that $\phi(t) = [\theta(t) - \theta^*] \to 0$ as $t \to \infty$ [32]. Results are plotted in Figure 3.6 for $0 \leq t \leq 100$. Convergence rate appears roughly the same for centralized and decentralized cases. Residual tracking errors are smaller (eventually zero) in the centralized case, which is to be expected since exact model-matching will occur. The centralized control law, however, requires twelve adaptation gains, compared to six in the decentralized case. In general, for the system composed of N interconnected subsystems, centralized controllers require $N(n + N)$ adaptation gains, where n is the total number of states. Decentralized controllers require only $n + N$ adaptation gains, and N sets of gain adaptation equations can be run in parallel. We are led to conclude that a decentralized adaptive controller has a far simpler algorithm than a centralized one, at the price of a relatively small decrease in performance.

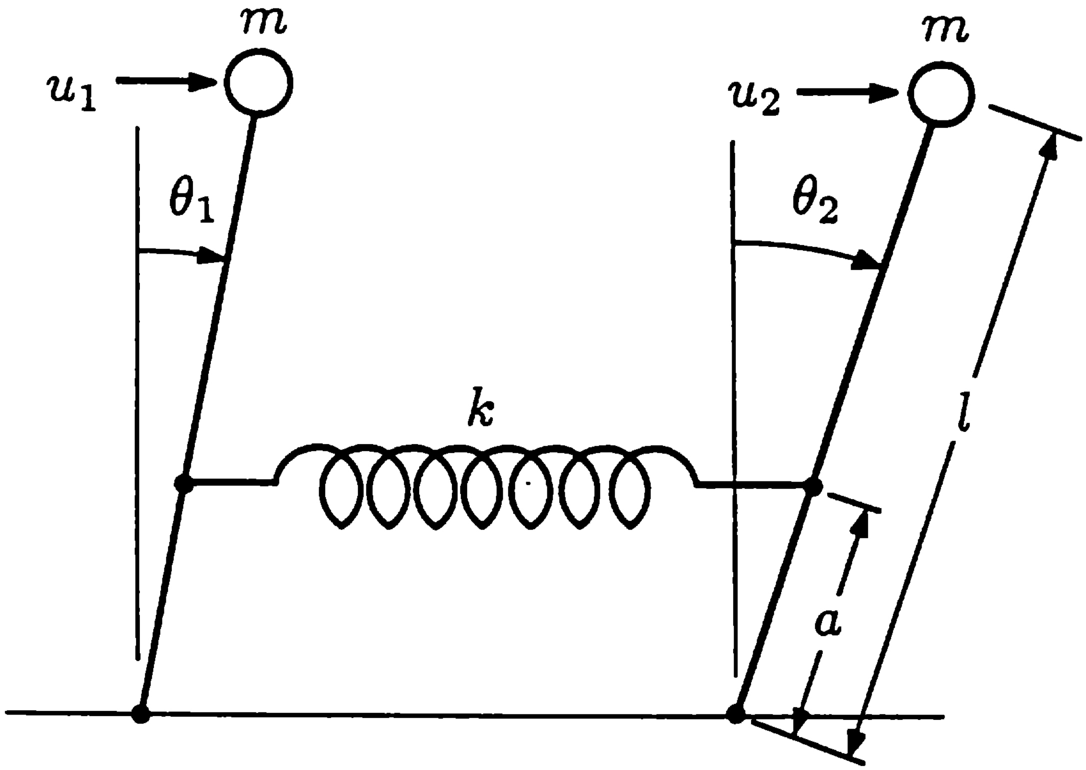

Figure 3.1. Interconnected penduli.

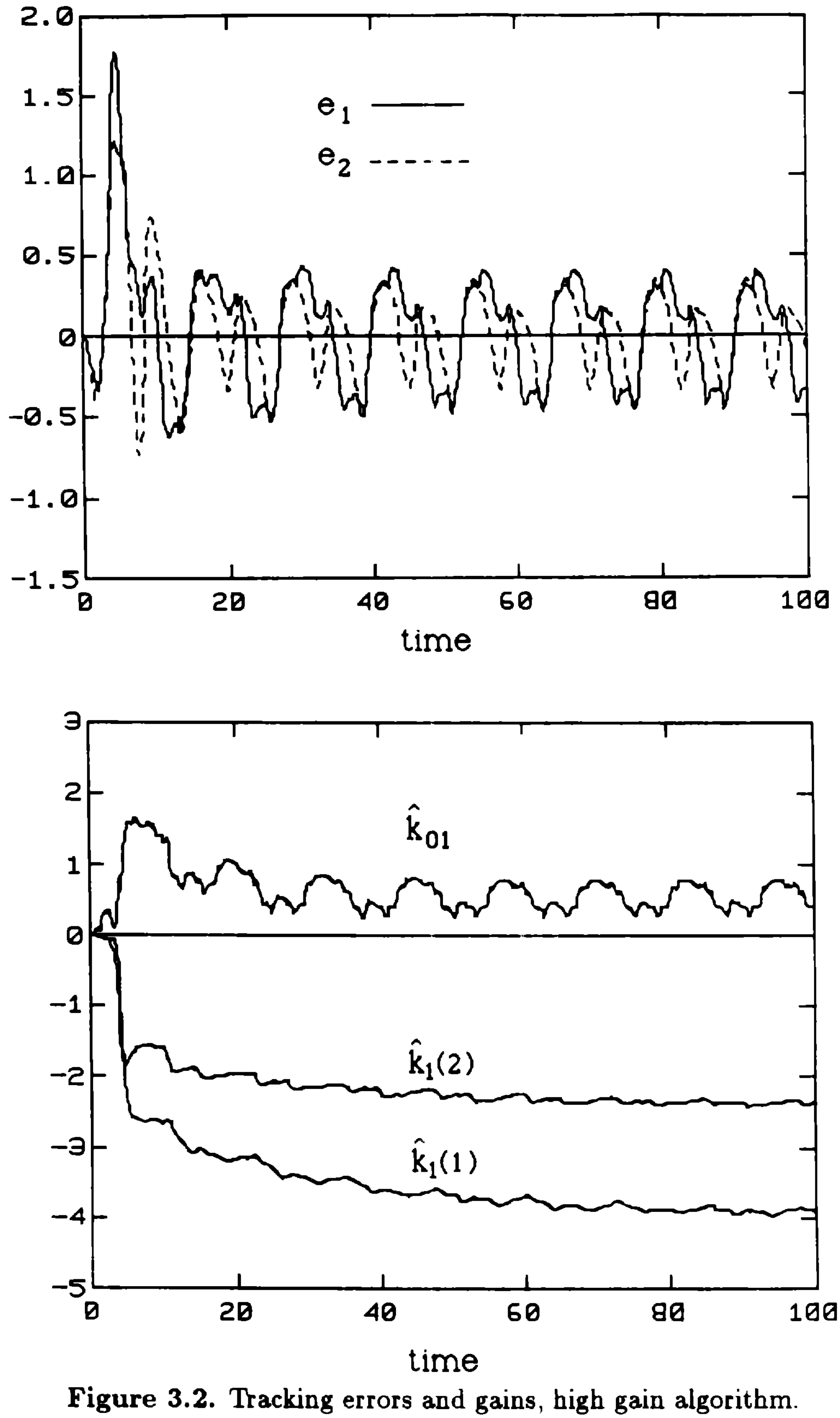

Figure 3.2. Tracking errors and gains, high gain algorithm.

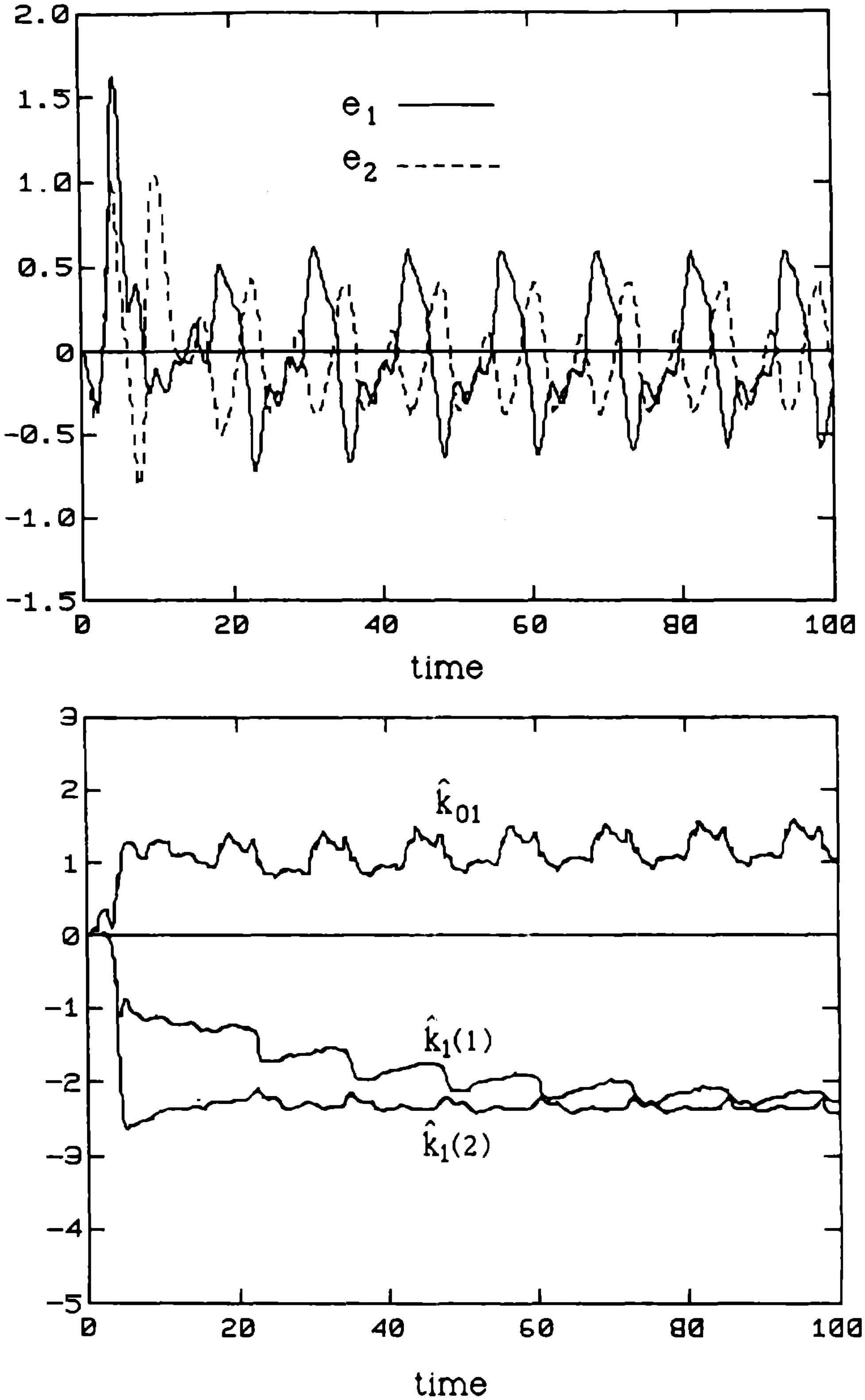

Figure 3.3. Tracking errors and gains, standard algorithm.

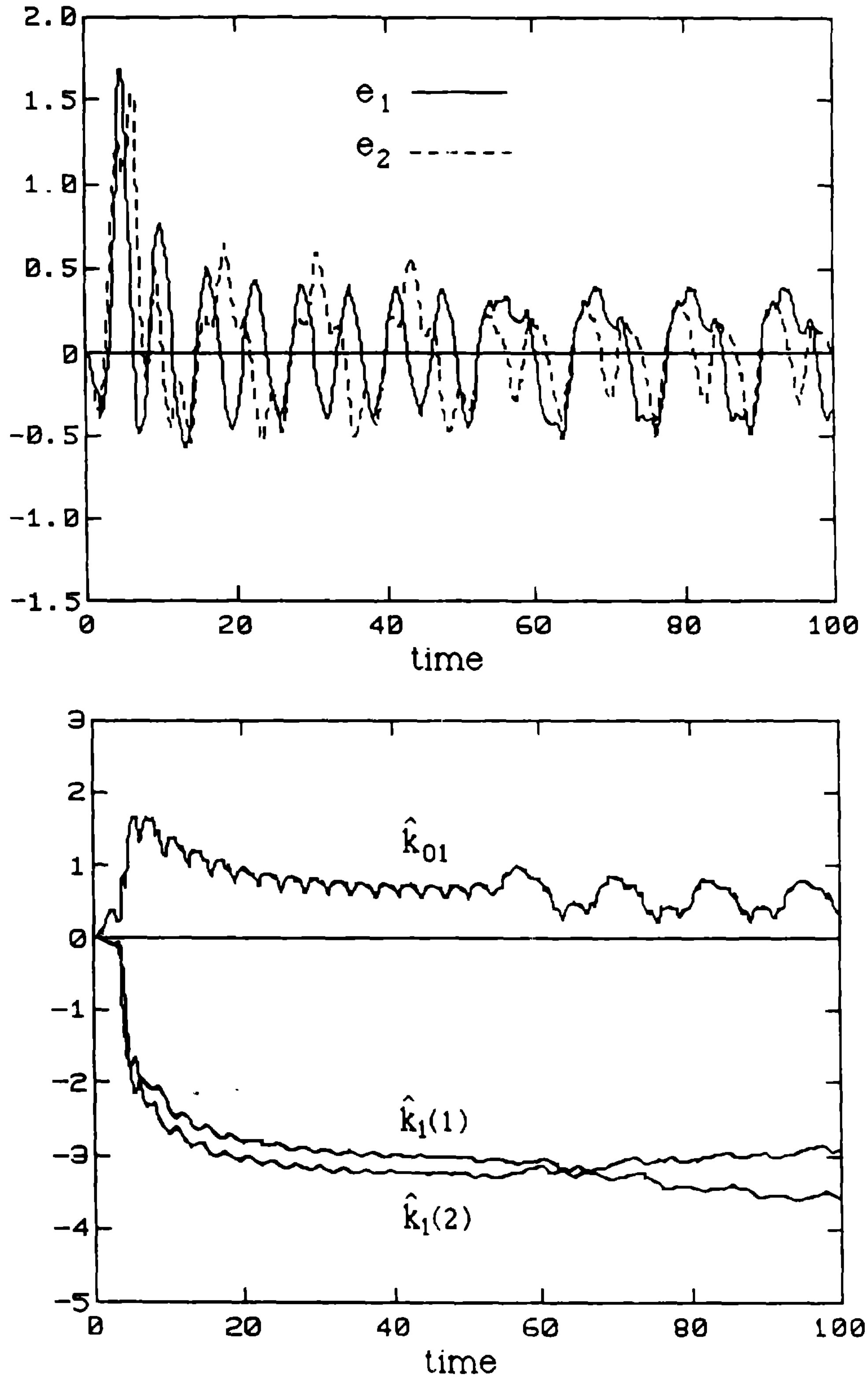

Figure 3.4. Time-varying interconnections, high gain algorithm.

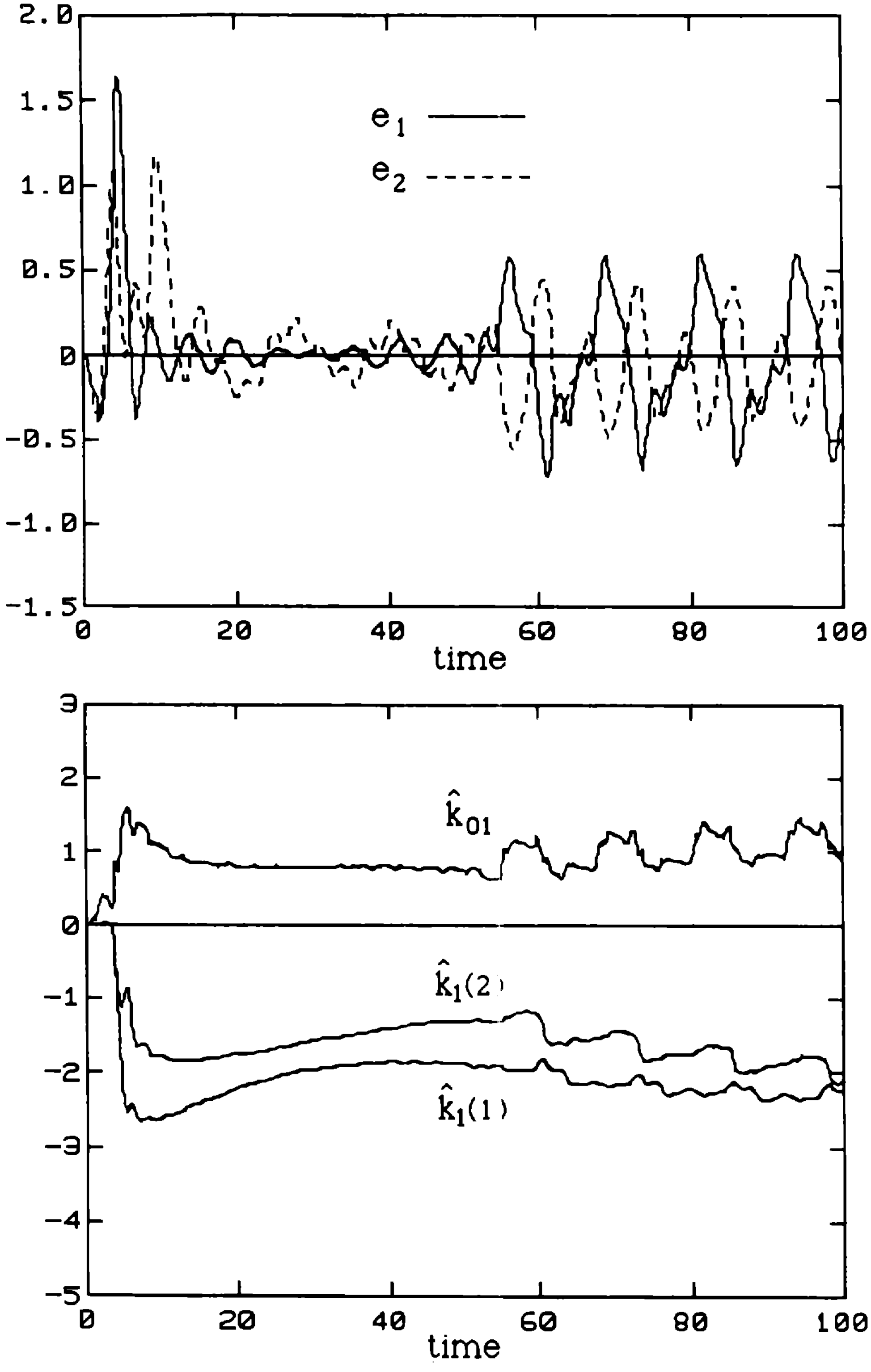

Figure 3.5. Time-varying interconnections, standard algorithm.

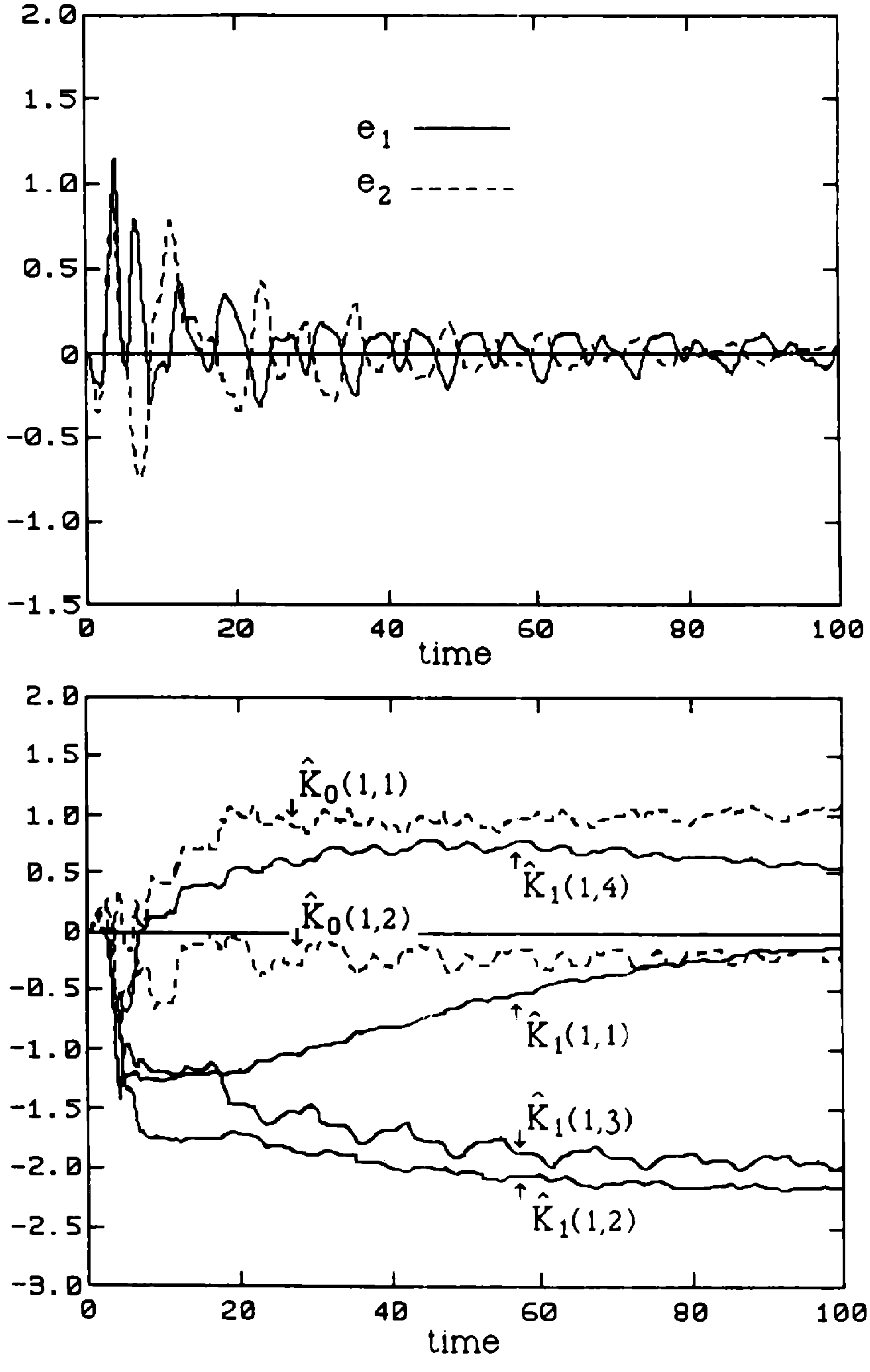

Figure 3.6. Tracking errors and gains, centralized control.

4

Utilizing Knowledge of the Subsystems

In deriving the adaptive control laws in Chapter 3. and in earlier work on decentralized adaptive control, there is a presumption of ignorance of subsystem dynamics. In modeling of large systems, however, the standard process of decomposing the system into a number of subsystems is often guided by our knowledge about the isolated parts of the system, and, as a consequence, the subsytems are often modeled well, while the essential uncertainty resides only in the interconnections.

Ignorance of subsystem dynamics may therefore be only an artifice which is used simply to make the established centralized adaptive algorithms applicable in a decentralized setting, rather than to reflect a genuine modeling uncertainty about the subsystems. Ideally, when the subsystems are known beforehand, the adaptive control law should be designed to exploit the knowledge of the subsystems, while adapting to compensate for unknown interconnection strengths.

We propose a new adaptive control algorithm which is applicable to decentralized systems where the subsystems are *known* and the interconnection strengths are *unknown*. With this new algorithm, we extend the decentralized adaptive control theory to the much broader class of systems described as interconnected multi-input, multi-output (MIMO) subsystems We also find that this new algorithm is simpler to implement than the earlier counterparts, in that it requires only one adapted parameter per subsystem The main theorem of this chapter shows that the algorithm regulates the overall system to zero, or tracks a given reference model with a finite error.

Consider the collection of MIMO subsystems

$$\begin{aligned} \mathbf{S}_i : \dot{x}_i &= A_i x_i + B_i u_i + P_i v_i \\ y_i &= C_i x_i \qquad\qquad\qquad i \in \boldsymbol{N} \\ w_i &= Q_i x_i \end{aligned} \tag{4.1}$$

where $x_i(t) \in \mathcal{R}^{n_i}$, $u_i(t) \in \mathcal{R}^{p_i}$, and $y_i(t) \in \mathcal{R}^{q_i}$ are the state, input, and output of the subsystem $\mathbf{S}_i$, and $v_i(t) \in \mathcal{R}^{m_i}$, and $w_i(t) \in \mathcal{R}^{l_i}$ are the interconnection inputs and outputs of $\mathbf{S}_i$ to and from other subsystems $\mathbf{S}_j$, $j \in \boldsymbol{N}$, at time $t \in \mathcal{R}$. We assume that the pair $\{A_i, B_i\}$ is controllable and the pair $\{A_i, C_i\}$ is observable.

The interconnection inputs and outputs, v_i and w_i, are related as

$$v_i = f_i(t, w) \tag{4.2}$$

and satisfy the conic sector bounds expressed in (2.6), repeated below.

$$||f_i(t, w)|| \leq \sum_{j=1}^{N} \xi_{ij} ||w_j|| \quad \forall (t, w) \in \mathcal{R} \times \mathcal{R}^m, \quad i \in \boldsymbol{N} \tag{4.3}$$

As before, we assume that the numbers ξ_{ij} are fixed but unknown, thus reflecting our uncertainty about the interconnections.

However, unlike before, we now assume that the matrices A_i, B_i, and C_i, which are associated with the subsystems, are known. Furthermore, we assume that the matrices P_i are factorable as

$$P_i = B_i P_i' \quad i \in \boldsymbol{N} \tag{4.4}$$

for some constant (but not necessarily known) matrices P_i'.

The objective will be for the closed loop subsystems to follow stable isolated reference models. We specify the reference models by first selecting local state feedback gain matrices $K_i \in \mathcal{R}^{p_i \times n_i}$ which would stabilize the local isolated subsystems. That is,

$$A_{mi} = A_i - B_i K_i, \quad i \in \boldsymbol{N} \tag{4.5}$$

is a stable matrix. The reference models are then described by

$$\begin{aligned} \mathbf{M}_i : \dot{x}_{mi} &= A_{mi} x_{mi} + B_i r_i \\ y_{mi} &= C_i x_{mi} \end{aligned} \qquad i \in \boldsymbol{N} \tag{4.6}$$

where $r_i : \mathcal{R} \rightarrow \mathcal{R}^{p_i}$ are bounded, piecewise continuous vector functions of time.

We now proceed with design of an adaptive control law. The first step is to choose any symmetric positive definite $n_i \times n_i$ matrix G_i and then find the symmetric positive definite solution H_i to the matrix Liapunov equation

$$A_{mi}^T H_i + H_i A_{mi} = -G_i, \quad i \in \boldsymbol{N} \tag{4.7}$$

A solution is guaranteed to exist since A_{mi} is stable. Now choose a symmetric, positive definite $p_i \times p_i$ matrix R_i and define $\bar{K}_i$ as

$$\bar{K}_i = R_i^{-1} B_i^T H_i, \quad i \in \boldsymbol{N} \tag{4.8}$$

Using K_i defined by (4.5) and $\bar{K}_i$ defined by (4.8), the proposed local feedback control law is

$$u_i = -K_i x_i - \alpha_i(t) \bar{K}_i e_i + r_i, \quad i \in \boldsymbol{N} \tag{4.9}$$

where $\alpha_i(t) \in \mathcal{R}$ is a time-variable *scalar* adaptation gain at time $t > t_0$ and $e_i(t) = x_i(t) - x_{mi}(t)$. We propose the following adaptation law for $\alpha(t)$

$$\dot{\alpha}_i = \gamma_i e_i^T \bar{K}_i^T R_i \bar{K}_i e_i - \gamma_i \sigma_i \alpha_i, \quad \alpha_i(t_0) > 0, \quad i \in \boldsymbol{N} \tag{4.10}$$

where γ_i and σ_i are given positive scalars.

To analyze the stability of the above control scheme, we form an error differential equation by subtracting (4.6) from (4.1). Together with (4.10), this describes the motion of the entire system:

$$\begin{aligned} \mathbf{S}_e : \dot{e}_i &= \left(A_{mi} - \alpha B_i \bar{K}_i\right) e_i + B_i P_i' v_i \\ \dot{\alpha}_i &= \gamma_i e_i^T \bar{K}_i^T R_i \bar{K}_i e_i - \gamma_i \sigma_i \alpha_i \end{aligned} \qquad i \in \boldsymbol{N} \tag{4.11}$$

Now define $e = (e_1^T, e_2^T, \ldots, e_N^T)^T$, $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_N)^T$ and denote the state of $\hat{\mathbf{S}}_e$ as $(e, \alpha) \in \mathcal{R}^n \times \mathcal{R}^N$.

(4.12) **THEOREM**. Solutions, $(e, \alpha)(t; t_0, e_0, \alpha_0)$ of the system $\hat{\mathbf{S}}_e$ are globally ultimately bounded.

PROOF. We use the Liapunov function

$$V(e, \alpha) = \sum_{i=1}^{N} e_i^T H_i e_i + \gamma_i^{-1}(\alpha_i - \alpha^*)^2 \tag{4.13}$$

where α^* is a constant positive parameter to be specified below. Taking the total time derivative with respect to (4.11) we get

$$\begin{aligned} \dot{V}(e, \alpha)_{(4.11)} = \sum_{i=1}^{N} \Big[& -e_i^T G_i e_i - 2\alpha^* e_i^T \bar{K}_i^T R_i \bar{K}_i e_i \\ & + 2 e_i^T \bar{K}_i^T R_i P_i' v_i - 2\sigma_i \alpha_i (\alpha_i - \alpha^*) \Big]. \end{aligned} \tag{4.14}$$

In view of the inequality (4.3), we can write

$$\begin{aligned} \dot{V}(e, \alpha)_{(4.11)} \le \sum_{i=1}^{N} \Big[& - e_i^T G_i e_i - 2\alpha^* \| R_i^{\frac{1}{2}} \bar{K}_i e_i \|^2 \\ & + 2\xi_{ij} \| R_i^{\frac{1}{2}} \bar{K}_i e_i \| \, \| R_i^{\frac{1}{2}} P_i' \| \, \| Q_j \| \left(\| e_j \| + \| x_{mj} \| \right) \\ & - 2\sigma_i \alpha_i^2 + 2\sigma_i \alpha_i \alpha^* \Big] \\ & \qquad \forall (t, e, \alpha) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^N. \end{aligned} \tag{4.15}$$

Now, completing the squares involving $\| R_i^{\frac{1}{2}} \bar{K}_i e_i \|$ and α_i, and utilizing the block notation introduced earlier, we have

$$\begin{aligned} \dot{V}(e, \alpha)_{(4.11)} \le & - \left[\lambda_{\min}(G) - \alpha^{*-1} \xi^2 \| R^{\frac{1}{2}} P' \|^2 \, \| Q \|^2 \right] \| e \|^2 - \sigma \| \alpha_i - \bar{\alpha}^* \|^2 \\ & + \alpha^{*-1} \xi^2 \| R^{\frac{1}{2}} P' \|^2 \, \| Q \|^2 \sup_t \| x_m(t) \|^2 + \sigma \alpha^{*2} \\ & \qquad \forall (t, e, \alpha) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^N \end{aligned} \tag{4.16}$$

where $\bar{\alpha}^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_N^*)^T$. Equation (4.16) can be written more compactly as

$$\begin{aligned} \dot{V}(e,\alpha)_{(4.11)} \le -\zeta \|e\|^2 - \sigma \|\alpha - \bar{\alpha}^*\|^2 + \eta \\ \forall (t,e,\alpha) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^N \end{aligned} \tag{4.17}$$

where the constants ζ and η are defined by

$$\begin{aligned} \zeta &= \lambda_{min}(G) - \alpha^{*-1}\xi^2 \|R^{\frac{1}{2}}P'\|^2 \|Q\|^2 \\ \eta &= \alpha^{*-1}\xi^2 \|R^{\frac{1}{2}}P'\|^2 \|Q\|^2 \sup_t \|x_m(t)\|^2 + \sigma\alpha^{*2}. \end{aligned} \tag{4.18}$$

By selecting a sufficiently large value for α^* so that ζ is a positive number, we have

$$\dot{V}(e,\alpha)_{(4.11)} \le -\mu V(e,\alpha) + \eta \quad \forall (t,\epsilon,\alpha) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^N \tag{4.19}$$

where

$$\mu \le \min\left[\lambda_{\max}^{-1}(H)\zeta,\ \min_i(\gamma_i)\sigma\right]. \tag{4.20}$$

From (4.19) it is clear that $V(e,\alpha)$ decreases monotonically along any solution of $\hat{\mathbf{S}}_e$ until the solution reaches the compact set

$$\Omega_f = \left\{(e,\alpha) \in \mathcal{R}^n \times \mathcal{R}^N : V(e,\alpha) \le V_f\right\} \tag{4.20}$$

where

$$V_f = \mu^{-1}\eta \tag{4.22}$$

Therefore, the solutions $(e,\alpha)(t;t_0,e_0,\alpha_0)$ of $\hat{\mathbf{S}}_e$ are globally ultimately bounded with respect to the bound V_f. **Q.E.D.**

(4.23) **REMARK**. The adaptation law (4.10) is considerably simpler than the ones presented earlier, since it involves *only one adapted parameter per subsystem* instead of the usual n_i adapted gains per subsystem. This is the main benefit of exploiting the knowledge of the subsystem dynamics.

Once again, it is not necessary to know or specify α^*, which is simply a parameter used in the proof above. The adapted gains α_i raise to whatever level is necessary to stabilize the overall system, regardless of the strength of the interconnections. The size of the residual set is adjusted through the choice of σ_i and γ_i.

(4.24) **EXAMPLE** We illustrate the applicability of this algorithm using the example from the last chapter. Using the same reference model and positive definite matrices G_i and H_i as before, we calculate the gain matrices

$$K_i = (\,2.0, \quad 2.0\,)\,, \quad \bar{K}_i = (\,0.5, \quad 0.5\,) \quad i \in \boldsymbol{N} \tag{4.23}$$

for use in the control law (3.9). The reference signal $r(t)$ is given by

$$\begin{aligned} r_1(t) &= \sin 0.2\pi t + \sin \pi t + \sin 2\pi t \\ r_2(t) &= 0 \end{aligned} \qquad t \geq t_0 = 0 \tag{4.24}$$

that is, it is desired, starting at $t = 0$, to move the first pendulum sinusoidally, while holding the second pendulum unmoving in a vertical position. At $t = 0$, the adapted gain, α_i is zero. We use $\gamma = 100$ and $\sigma = 0.0001$ in the adaptation law (4.10).

Simulation results are shown in Figure 4.1. Pendulum positions closely follow the reference model, with accuracy improving over time. The gains $\alpha_i(t)$ adjust generally upward during a transient period, and then oscillate around a fixed positive value in steady state.

For comparison, we simulated the same system in closed-loop, but without adaptation. Setting $\alpha_i(t) = 0$, $i = \{1, 2\}$, $t \geq 0$, we get the results shown in Figure 4.2. Notice that, without adaptation, a large amount of the signal intended to drive the first pendulum couples into the second pendulum through the connecting spring, whereas with adaptation, local gains increase in order to reduce the coupling effect.

This adaptive control law is the simplest of the ones described so far, since only two parameters are adapted in the overall system, as compared to six for the decentralized adaptive controller described in Chapter 3, and twelve for the centralized adaptive controller. For a system with N subsystems, there are N adaptatation gains, and the N adaptation gain equations can be run in parallel.

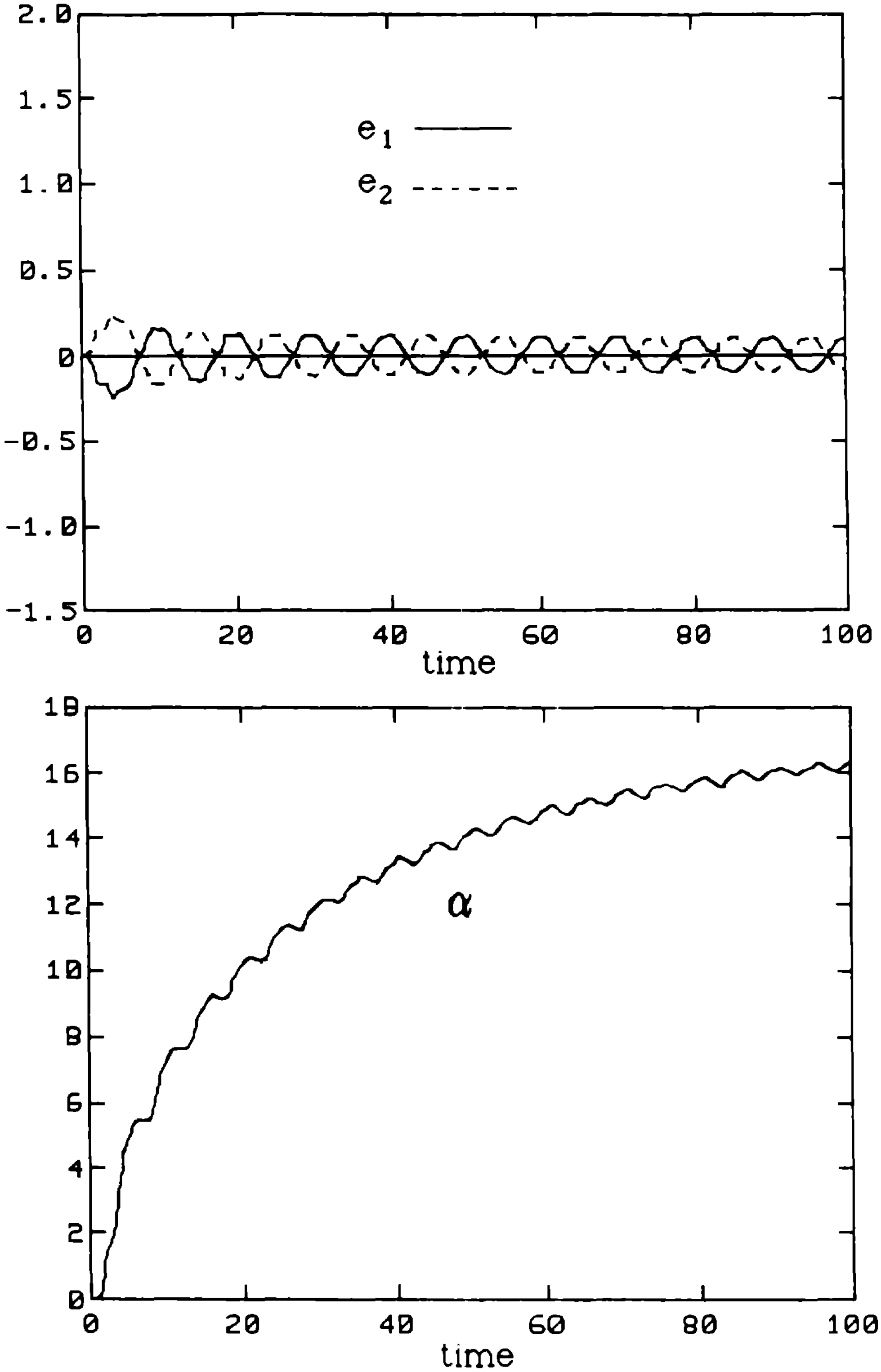

Figure 4.1. Tracking errors and gains, known subsystem scheme.

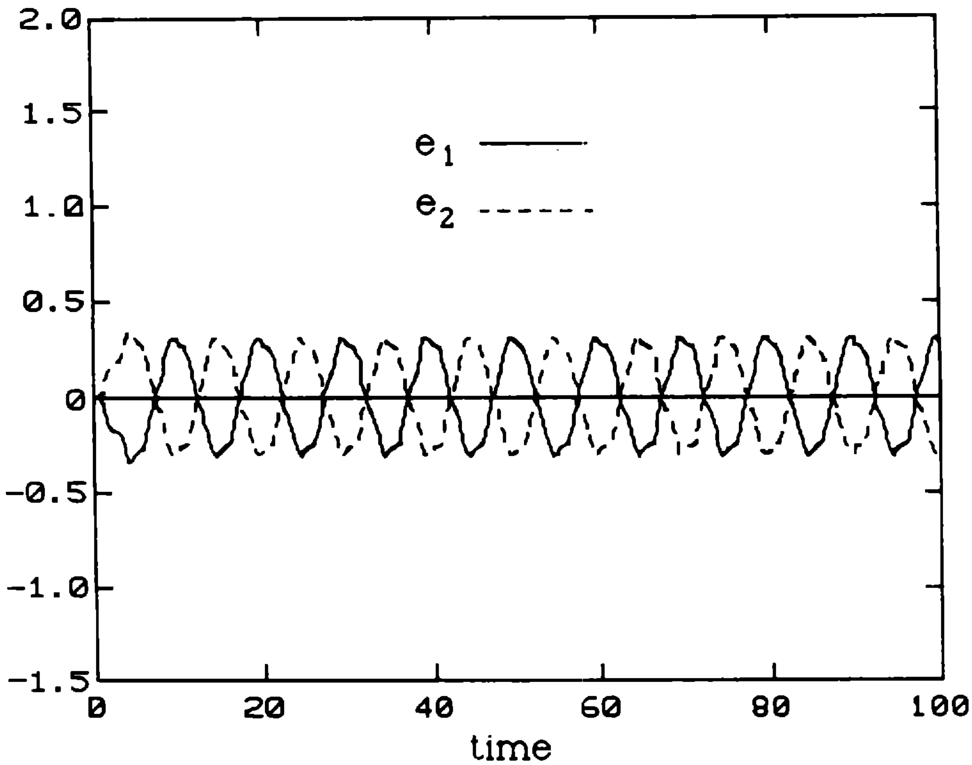

Figure 4.2. Tracking error without adaptation.

5
Output Feedback

In this chapter we assume that only the outputs y_i of the subsystems are available for use in feedback control. It is therefore necessary to construct a set of *dynamic* controllers which will stabilize the overall system. The well known dynamic adaptive controller for centralized systems was introduced in [11], and proven stable in [12]. In later work, the dynamic controller was reproduced for each subsystem in a decentralized control context in [22]. We use the same type of controller, with a slight modification, in the present decentralized high-gain feedback scheme.

Each subsystem controller consists of a precompensator, $\mathbf{C}_i^p$ and a feedback compensator $\mathbf{C}_i^f$ leading to the open-loop subsystem description

$$\begin{aligned} \mathbf{S}_i : \dot{x}_i &= A_i x_i + b_i u_i + P_i v_i \\ y_i &= c_i^T x_i \\ w_i &= Q_i x_i \qquad\qquad i \in N \\ \mathbf{C}_i^p : \dot{z}_{pi} &= F_i z_{pi} + g_i u_i \\ \mathbf{C}_i^f : \dot{z}_{fi} &= F_i z_{fi} + g_i y_i \end{aligned} \tag{5.1}$$

where $x_i(t) \in \mathcal{R}^{n_i}$, $z_{pi}(t) \in \mathcal{R}^{n_i-1}$, and $z_{fi}(t) \in \mathcal{R}^{n_i-1}$, are the states of $\mathbf{S}_i$, $\mathbf{C}_i^p$ and $\mathbf{C}_i^f$ at time $t \in \mathcal{R}$, $u_i(t) \in \mathcal{R}^{p_i}$, and $y_i(t) \in \mathcal{R}^{q_i}$ are the input, and output of the subsystem $\mathbf{S}_i$, and $v_i(t) \in \mathcal{R}^{m_i}$, and $w_i(t) \in \mathcal{R}^{l_i}$ are the interconnection inputs and outputs of $\mathbf{S}_i$ to and from other subsystems $\mathbf{S}_j$, $j \in N$, at time $t \in \mathcal{R}$. We assume that the pair $\{A_i, b_i\}$ is controllable and the pair $\{A_i, c_i^T\}$ is observable.

The interconnection inputs and outputs, v_i and w_i, are related as

$$v_i = f_i(t, w), \quad i \in N \tag{5.2}$$

and satisfy the conic sector bounds expressed in (2.6), repeated below.

$$\|f_i(t,w)\| \leq \sum_{j=1}^{N} \xi_{ij}\, \|w_j\| \qquad \forall (t,w) \in \mathcal{R} \times \mathcal{R}^m, \quad i \in \boldsymbol{N} \tag{5.3}$$

As before, we assume that the numbers ξ_{ij} are fixed but unknown, thus reflecting our uncertainty about the interconnections.

For stabilizability, we assume that there exist subsets $\boldsymbol{K} \subset \boldsymbol{N}$ and $\boldsymbol{K}' = \boldsymbol{N} - \boldsymbol{K}$ such that

$$\begin{aligned} P_i &= b_i p_i^T, \quad i \in \boldsymbol{K} \\ Q_i &= q_i c_i^T, \quad i \in \boldsymbol{K}' \end{aligned} \tag{5.4}$$

for some constant vectors $p_i \in \mathcal{R}^{m_i}$, $q_i \in \mathcal{R}^{l_i}$. This type of large scale system was studied in [7] and proven to be stabilizable by fixed decentralized feedback. For ease in notation, we will number the subsystems such that $\boldsymbol{K} = \{1, 2, \ldots, K\}$, $K \leq N$, and $\boldsymbol{K}' = \{K+1, K+2, \ldots, N\}$.

Our proposed adaptive decentralized control law is given by

$$u_i = \theta_i^T \nu_i, \qquad i \in \boldsymbol{N} \tag{5.5}$$

where

$$\nu_i = (\tilde{y}_i, z_{pi}^T, z_{fi}^T, r_i)^T \tag{5.6}$$

is a vector of available signals, and

$$\theta_i = (\hat{d}_{fi}, \hat{c}_{pi}^T, \hat{c}_{fi}^T, \hat{k}_{0i})^T \tag{5.7}$$

is a vector of adaptation gains. In (5.6), $r_i(t)$ is the external reference input, which is a uniformly bounded, piecewise continuous function of t, and $\tilde{y}_i = y_i - y_{mi}$ is the model output following error. Stable reference models $\mathbf{M}_i$ are specified as before:

$$\begin{aligned} \mathbf{M}_i : \dot{x}_{mi} &= A_{mi} x_{mi} + b_i r_i \\ y_{mi} &= c_{mi}^T x_{mi} \end{aligned} \qquad i \in \boldsymbol{N}. \tag{5.7}$$

Now define the transfer functions

$$\begin{aligned} \mathbf{S}_i &: \frac{y_i(s)}{u_i(s)} = \varphi_i(s) = \kappa_i \frac{\beta_i(s)}{\alpha_i(s)} \\ \hat{\mathbf{S}}_i &: \frac{y_i(s)}{r_i(s)} = \hat{\varphi}_i(s,\theta) = \hat{\kappa}_i(\theta)\frac{\hat{\beta}_i(s,\theta)}{\hat{\alpha}_i(s,\theta)} \qquad i \in N \\ \mathbf{M}_i &: \frac{y_{mi}(s)}{r_i(s)} = \varphi_{mi}(s) = \kappa_{mi}\frac{\beta_{mi}(s)}{\alpha_{mi}(s)} \end{aligned} \tag{5.8}$$

which describe the open loop plant, closed loop plant, and the reference model, respectively. The linear time-invariant system transfer function $\hat{\varphi}_i(s,\theta)$ is defined for fixed values of $\theta \in \mathcal{R}^{2n_i+2}$. In order for a decentralized adaptive control design to be feasible, the following conditions must be met:

(*i*) ***The plant is minimum phase***, that is, the monic polynomial $\beta_i(s)$ is Hurwitz (all zeros have negative real parts).

(*ii*) ***The plant has relative degree one***, that is the degree of $\beta_i(s)$ is $n_i - 1$, where n_i is the degree of $\alpha_i(s)$.

(*iii*) ***The sign of κ_i is known***. Without loss of generality, we assume that $\kappa_i > 0$.

In addition to the above conditions on the open loop plant, we must chose reference models that are ***stricly positive real***, which means that $\alpha_{mi}(s)$ and $\beta_{mi}(s)$ are Hurwitz, and $\mathrm{Re}\{\varphi_{mi}(j\omega)\} > 0$ for all $\omega \in [0\ \infty)$.

Under these conditions, it has been established [31] that there exists a unique $\theta_i^* = (d_{fi}^*, c_{pi}^*, c_{fi}^*, k_{0i}^*)^T$, such that the closed loop isolated subsystem $\hat{\mathbf{S}}_i$ has the same input/output behavior as the reference model $\mathbf{M}_i$, that is, $\hat{\varphi}_i(s,\theta_i^*) = \varphi_{mi}(s)$, $i \in N$.

In fact, however, we do not know θ_i^*, so we use its estimate, $\theta_i(t)$ in the control law (5.5). The proposed decentralized adaptation law for $\theta_i(t)$ is

$$\dot{\theta}_i = -\Gamma_i(\sigma\theta_i + \nu_i\tilde{y}_i), \quad i \in N \tag{5.9}$$

where Γ_i is a constant, symmetric positive definite matrix of dimension $(3n_i - 2) \times (3n_i - 2)$, and σ is a constant positive scalar.

With $\theta_i(t)$ different from θ_i^*, the closed loop isolated subsystem acts like the reference model, but with the added disturbance input $\phi_i^T(t)\nu_i(t)$ where $\phi_i(t) = \theta_i(t) - \theta_i^*$. The closed loop interconnected system is thus described by

$$\begin{aligned}
\hat{\mathbf{S}}_i : \quad \dot{\hat{x}}_i &= \hat{A}_i(\theta_i^*)\hat{x}_i + \hat{b}_i\phi_i^T\nu_i + \hat{b}_i r_i + \hat{P}_i v_i - b_{pi}d_{fi}^* y_{mi} \\
y_i &= \hat{c}_i^T\hat{x}_i \qquad\qquad i \in \boldsymbol{N} \qquad\qquad (5.10)\\
w_i &= \hat{Q}_i\hat{x}_i
\end{aligned}$$

where $\hat{x}_i = (x_i^T, z_{pi}^T, z_{fi}^T)^T$, and

$$\begin{aligned}
\hat{A}_i(\theta_i^*) &= \begin{pmatrix} A_i + b_i d_i^* c_i^T & b_i c_{pi}^{*T} & b_i c_{fi}^{*T} \\ g_i d_i^* c_i^T & F_i + g_i c_{pi}^{*T} & g_i c_{fi}^{*T} \\ g_i c_i^T & 0 & F_i \end{pmatrix} \\
\hat{b}_i &= (b_i^T, g_i^T, 0)^T \\
\hat{c}_i^T &= (c_i^T, 0, 0) \qquad\qquad (5.11)\\
\hat{P}_i &= (P_i^T, 0, 0)^T \\
\hat{Q}_i &= (Q_i, 0, 0)
\end{aligned}$$

The triple $\{\hat{A}_i(\theta_i^*), \hat{b}_i, \hat{c}_i^T\}$ is a non-minimal realization of the reference model:

$$\begin{aligned}
\hat{\mathbf{M}}_i : \dot{\hat{x}}_{mi} &= \hat{A}_i(\theta_i^*)\hat{x}_{mi} + \hat{b}_i r_i \\
y_{mi} &= \hat{c}_{mi}^T\hat{x}_{mi}
\end{aligned} \qquad i \in \boldsymbol{N}. \qquad (5.12)$$

The reference model is strictly positive real, so that, from the Kalman-Yacubovich lemma (*e.g.*, [10]) it follows that $\{\hat{A}_i(\theta_i^*), \hat{b}_i, \hat{c}_i^T\}$ satisfies the equations

$$\begin{aligned}
\hat{A}_i^T(\theta_i^*)\hat{H}_i + \hat{H}_i\hat{A}_i(\theta_i^*) &= -l_i l_i^T - \epsilon_i\hat{L}_i \\
\hat{H}_i\hat{b}_i &= \hat{c}_i
\end{aligned} \qquad i \in \boldsymbol{N}, \qquad (5.13)$$

for some constant, symmetric positive definite matrices, $\hat{H}_i$ and $\hat{L}_i$, some constant vector, $l_i \in \mathcal{R}^{3n_i-2}$, and some constant scalar, $\epsilon_i > 0$.

An equation for tracking error $\hat{e}_i(t) = \hat{x}_i(t) - \hat{x}_{mi}(t)$ can be derived by subtracting (5.12) from (5.10),

$$\begin{aligned}
\hat{\mathbf{S}}_{ei} : \dot{\hat{e}}_i &= \hat{A}_i(\theta_i^*)\hat{e}_i + \hat{b}_i\phi_i^T\nu_i + \hat{P}_i v_i - \hat{b}_{pi}d_{fi}^* y_{mi} \\
\tilde{y}_i &= \hat{c}_i^T\hat{e}_i \qquad\qquad i \in \boldsymbol{N}. \qquad\qquad (5.14)\\
w_i &= \hat{Q}_i(\hat{e}_i + \hat{x}_{mi})
\end{aligned}$$

The decentralized stabilizabilty conditions (5.4) are stated with respect to the open loop systems $\mathbf{S}$. The closed loop system $\hat{\mathbf{S}}$ with the local dynamic compensators, is a structurally different large scale system, so we must restate the stabilizability conditions in terms of the new structure. The condition

$$Q_i = q_i \hat{c}_i^T, \quad i \in \boldsymbol{K}' \tag{5.15}$$

follows immediately, however it is *not* true that $\hat{P}_i = \hat{b}_i p_i^T$, $i \in \boldsymbol{K}$. This is because interconnection disturbances that enter at the input to the plant, are not available signals for input to the precompensator as well (see Figure 5.1a). We can, however, *reflect* the disturbances to the input of the closed loop system as shown in Figure 5.1b.

From (5.1) we calculate the transfer function for the *open loop* precompensator as

$$\mathbf{C}_i^p : \varphi_{pi}(s) = c_{pi}^{*T}(sI - F_i)^{-1} g_i, \quad i \in \boldsymbol{K}. \tag{5.16}$$

When the loop is closed, the precompensator acts as a prefilter for the feedback, reference, and adaptation error signals. This prefilter has the transfer function

$$\hat{\mathbf{C}}_i^p : \hat{\varphi}_{pi}(s) = [1 - \varphi_{pi}(s)]^{-1} = c_{pi}^{*T}(sI - F_i - g_i c_{pi}^{*T})^{-1} g_i, \quad i \in \boldsymbol{K}. \tag{5.17}$$

In [31] it is shown that

$$\hat{\varphi}_{pi}(s) = \frac{\beta_{mi}(s)}{\beta_i(s)} \quad i \in \boldsymbol{K}, \tag{5.18}$$

therefore, since both $\beta_i(s)$ and $\beta_{mi}(s)$ are assumed Hurwitz, we have that $\hat{\varphi}_{pi}(s)$ and $\hat{\varphi}_{pi}^{-1}(s)$ are stable transfer functions.

For each subsystem $\mathbf{S}_{ei}$, $i \in \boldsymbol{K}$, we reflect the interconnection signal $p_i^T v_i$ to the input of the closed loop subsystem by disconnecting it from the input of the plant, filtering it through the *inverse* of the prefilter $\hat{\mathbf{C}}_i^p$ then feeding the resulting filtered signal to the input of the closed loop subsystem, as shown in Figure 5.1b. In doing this, we have introduced K new subsystems into the analysis, whose transfer functions are $\hat{\varphi}_{pi}^{-1}(s)$ and which have the state space representation

$$\begin{aligned} \mathbf{F}_i : \dot{z}_i &= F_i z_i + g_i p_i^T v_i; \quad z_i(t_0) = 0, \\ \eta_i &= -c_{pi}^{*T} z_i + p_i^T v_i, \end{aligned} \qquad i \in \boldsymbol{K}, \tag{5.19}$$

where $z_i \in \mathcal{R}^{n_i - 1}$ is the state, $p_i^T v_i$ is the input, and $\eta_i \in \mathcal{R}$ is the output of $\mathbf{F}_i$. Since F_i is a stable matrix by design, the subsystems $\mathbf{F}_i$ have, associated with them, the Liapunov equations

$$F_i^T H_i' + H_i' F_i = -G_i', \quad i \in \boldsymbol{K}, \tag{5.20}$$

where both H_i' and G_i' are positive definite $n_i - 1 \times n_i - 1$ matrices.

With this modification, the closed loop error systems $\hat{\mathbf{S}}_{ei}$, $i \in \boldsymbol{K}$ are now described by

$$\hat{\mathbf{S}}_{ei} : \dot{\hat{e}}_i = \hat{A}_i(\theta^*)\hat{e}_i + \hat{b}_i \phi_i^T \nu_i + \hat{b}_i \eta_i - b_{pi} d_{fi}^* y_{mi}, \quad i \in \boldsymbol{K}. \tag{5.21}$$

We are now in a position to fully take advantage of the interconnection structure, as originally stated by conditions (5.4).

In summary, the differential equations for the large-scale, interconnected, adaptively controlled system are:

$$\begin{aligned} \hat{\mathbf{S}}_e : \dot{\hat{e}}_i &= \hat{A}_i \hat{e}_i + \hat{b}_i \left[\phi_i^T \nu_i + \eta_i - d_{fi}^* y_{mi}\right], \quad i \in \boldsymbol{K} \\ \dot{z}_i &= F_i z_i + g_i p_i^T v_i, \qquad i \in \boldsymbol{K} \\ \dot{\hat{e}}_i &= \hat{A}_i \hat{e}_i + \hat{b}_i \left[\phi_i^T \nu_i - d_{fi}^* y_{mi}\right] + \hat{P}_i v_i, \quad i \in \boldsymbol{K}' \\ \dot{\phi}_i &= -\Gamma_i \sigma(\phi_i + \theta_i^*) - \Gamma_i(\hat{c}_i^T \hat{e}_i)\nu_i, \quad i \in \boldsymbol{N} = \boldsymbol{K} + \boldsymbol{K}' \end{aligned} \tag{5.22}$$

with interconnections given by

$$\begin{aligned} \eta_i &= c_{pi}^{*T} z_{pi} + p_i^T v_i, \quad i \in \boldsymbol{K} \\ w_i &= \begin{cases} \hat{Q}_i(\hat{e}_i + \hat{x}_{mi}), & i \in \boldsymbol{K} \\ \hat{q}_i \hat{c}_i^T(\hat{e}_i + \hat{x}_{mi}), & i \in \boldsymbol{K}' \end{cases} \\ v_i &= f_i(t, w), \qquad i \in \boldsymbol{N} \end{aligned} \tag{5.23}$$

We denote the state of the overall system $\mathbf{S}_e$ as $(\hat{e}, z, \phi) \in \Omega$ where

$$\begin{aligned}
\Omega &= \mathcal{R}^{(3n-2N)} \times \mathcal{R}^k \times \mathcal{R}^{2n} \\
k &= \sum_{i=1}^{K} n_i \\
\hat{e} &= (\hat{e}_1^T, \hat{e}_2^T, \ldots, \hat{e}_N^T)^T \\
\phi &= (\phi_1^T, \phi_2^T, \ldots, \phi_N^T)^T \\
z &= (z_1^T, z_2^T, \ldots, z_K^T)^T
\end{aligned} \tag{5.24}$$

(5.25) **THEOREM.** The solutions $(\hat{e}, z, \phi)(t; t_0, \hat{e}_0, z_0, \phi_0)$ of the system $\mathbf{S}_e$ are globally ultimately bounded.

PROOF. Define the function $V : \Omega \to \mathcal{R}_+$ as

$$V(\hat{e}, z, \phi) = \sum_{i \in N} \delta_i \left[k_{0i}^* \hat{e}_i^T \hat{H}_i \hat{e}_i + (\phi_i + \rho\bar{\theta}_i)^T \Gamma_i^{-1} (\phi_i + \rho\bar{\theta}_i) \right] + \sum_{i \in K} \delta_i' z_i^T H_i' z_i \tag{5.26}$$

where $\bar{\theta}_i = (1, 0, \ldots, 0)^T$ are vectors having the same dimension as θ_i for $i \in \boldsymbol{N}$, and ρ, δ_i and δ_i' are as yet unspecified positive constants. Taking the time derivative of $V(\hat{e}, z, \phi)$ with respect to (5.22), we use the fact that $\bar{\theta}_i^T \nu_i = \hat{c}_i^T \hat{e}_i$ to get

$$\begin{aligned}
\dot{V}_{(5.22)}(\hat{e}, z, \phi) = &\sum_{i \in N} \delta_i \big[-\hat{e}_i^T (\hat{l}_i \hat{l}_i^T + \epsilon \hat{L}_i) \hat{e}_i - 2\hat{e}_i^T \hat{c}_i d_{fi}^* y_{mi} \\
&- 2\rho \hat{e}_i^T \hat{c}_i \hat{c}_i^T \hat{e}_i - 2\sigma (\phi_i + \rho\bar{\theta}_i)^T \theta_i \big] \\
&+ \sum_{i \in K} \big[-\delta_i' z_i^T G_i' z_i + 2\delta_i' z_i^T H_i' g_i p_i^T v_i + 2\delta_i \hat{e}_i^T \hat{c}_i \eta_i \big] \\
&+ \sum_{i \in K'} 2\delta_i \hat{e}_i^T \hat{H}_i \hat{P}_i v_i .
\end{aligned} \tag{5.27}$$

Substituting (5.23) and applying inequality (5.3), we get

$$
\begin{aligned}
\dot{V}_{(5.22)}(\hat{e}, z, \phi) \leq & \sum_{i\in N} \delta_i \left(-\hat{e}_i^T G_i \hat{e}_i - 2\rho \left|\hat{c}_i^T \hat{e}_i\right|^2 - 2\sigma \left\|\phi_i + \rho\bar{\theta}_i\right\|^2 + 2\sigma \left\|\theta_i^* - \rho\bar{\theta}_i\right\|^2\right) \\
& - \sum_{i\in K} \delta_i' z_i^T G_i' z_i \\
& + \sum_{i\in K} 2\delta_i \left|\hat{c}_i^T \hat{e}_i\right| \left[\sum_{j\in K} (\xi \|\hat{e}_j\| + \xi \|z_j\| + \chi) + \sum_{j\in K'} \left(\xi \left|\hat{c}_j^T \hat{e}_j\right| + \chi\right)\right] \\
& + \sum_{i\in K'} 2\delta_i \|\hat{e}_i\| \left[\sum_{j\in K} (\xi \|\hat{e}_j\| + \chi) + \sum_{j\in K'} \left(\xi \left|\hat{c}_j^T \hat{e}_j\right| + \chi\right)\right] \\
& + \sum_{i\in K} 2\delta_i' \|z_i\| \left[\sum_{j\in K} (\xi \|\hat{e}_j\| + \chi) + \sum_{j\in K'} \left(\xi \left|\hat{c}_j^T \hat{e}_j\right| + \chi\right)\right] \\
& \qquad \forall (\hat{e}, z, \phi) \in \Omega
\end{aligned} \tag{5.28}
$$

where

$$
\begin{aligned}
G_i &= \hat{l}_i \hat{l}_i^T + \epsilon_i \hat{L}_i \\
\xi &= \max_{i,j\in N} \max\{\|p_i\| \, \xi_{ij} \|\hat{Q}_j\|, \|p_i\| \, \xi_{ij} \, \|q_j\|, \left\|c_{pi}^*\right\|, \|\hat{H}_i \hat{P}_i\| \xi_{ij} \|\hat{Q}_j\|, \\
& \quad \|\hat{H}_i \hat{P}_i\| \xi_{ij} \, \|q_j\|, \left\|H_i' g_i p_i^T\right\| \xi_{ij} \|\hat{Q}_j\|, \left\|H_i' g_i p_i^T\right\| \xi_{ij} \, \|q_j\|, \left\|d_{ji}^* \hat{c}_i\right\|\} \\
\chi &= \xi \max_{i\in N} \left\{ \sup_{t\in\mathcal{R}_+} \|\hat{x}_{mi}(t)\| \right\}.
\end{aligned} \tag{5.29}
$$

It is useful to examine the block structure of (5.28). Define

$$
\begin{aligned}
\mathbf{e}_1 &= (\|\hat{e}_1\|, \|\hat{e}_2\|, \dots, \|\hat{e}_K\|)^T \\
\mathbf{e}_2 &= (\|\hat{e}_{K+1}\|, \|\hat{e}_{K+2}\|, \dots, \|\hat{e}_N\|)^T \\
\mathbf{e}_3 &= (\|z_1\|, \|z_2\|, \dots, \|z_K\|)^T \\
\tilde{\mathbf{y}}_1 &= (|\hat{c}_1^T \hat{e}_1|, |\hat{c}_2^T \hat{e}_2|, \dots, |\hat{c}_K^T \hat{e}_K|)^T \\
\tilde{\mathbf{y}}_2 &= (|\hat{c}_{K+1}^T \hat{e}_{K+1}|, |\hat{c}_{K+2}^T \hat{e}_{K+2}|, \dots, |\hat{c}_N^T \hat{e}_N|)^T \\
\mathbf{G}_1 &= \mathrm{diag}\{\lambda_{\min}(G_1), \lambda_{\min}(G_2), \dots, \lambda_{\min}(G_K)\} \\
\mathbf{G}_2 &= \mathrm{diag}\{\lambda_{\min}(G_{K+1}), \lambda_{\min}(G_{K+2}), \dots, \lambda_{\min}(G_N)\} \\
\mathbf{G}_3 &= \mathrm{diag}\{\lambda_{\min}(G'_1), \lambda_{\min}(G'_2), \dots, \lambda_{\min}(G'_N)\} \\
\mathbf{D} &= \mathrm{diag}\{\delta_1, \delta_2, \dots, \delta_K, \delta_{K+1}, \dots, \delta_N, \delta'_1, \dots, \delta'_K\}.
\end{aligned}
\tag{5.30}
$$

Also, let $\bar{\delta} = \max\{\delta_1, \delta_2, \dots, \delta_K, \delta_{K+1}, \dots, \delta_N, \delta'_1, \dots, \delta'_K\}$ and $\underline{\delta} = \min\{\delta_1, \delta_2, \dots, \delta_K, \delta_{K+1}, \dots, \delta_N, \delta'_1, \dots, \delta'_K\}$. The structure of (5.28) can now be seen as

$$
\begin{aligned}
\dot{V}_{(5.22)}(\hat{e}, z, \phi) \le &-(\mathbf{e}_1^T, \mathbf{e}_2^T, \mathbf{e}_3^T)(\mathbf{DL} + \mathbf{L}^T\mathbf{D}) \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{pmatrix} - 2\underline{\delta}\sigma \left\|\phi + \rho\bar{\theta}\right\|^2 \\
&- 2\underline{\delta}\rho(\tilde{\mathbf{y}}_1^T, \tilde{\mathbf{y}}_2^T) \begin{pmatrix} \tilde{\mathbf{y}}_1 \\ \tilde{\mathbf{y}}_2 \end{pmatrix} + 2\bar{\delta}\xi(\mathbf{e}_1^T, \mathbf{e}_2^T, \mathbf{e}_3^T)\mathbf{M} \begin{pmatrix} \tilde{\mathbf{y}}_1 \\ \tilde{\mathbf{y}}_2 \end{pmatrix} \\
&+ 2\bar{\delta}\chi\mathbf{v}^T \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{pmatrix} + 2\bar{\delta}\sigma \left\|\theta^* - \rho\bar{\theta}\right\|^2 \\
&\forall(\hat{e}, z, \phi) \in \Omega
\end{aligned}
\tag{5.31}
$$

where

$$
-\mathbf{L} = \begin{pmatrix} -\mathbf{G}_1 & 0 & 0 \\ \xi\mathbf{F}_{21} & -\mathbf{G}_2 & 0 \\ \xi\mathbf{F}_{31} & \xi\mathbf{F}_{32} & -\mathbf{G}_3 \end{pmatrix}
$$

and $\mathbf{F}_{21}$, $\mathbf{F}_{31}$, $\mathbf{F}_{32}$, and $\mathbf{M}$ are block matrices of appropriate size containing all ones, and $\mathbf{v} \in \mathcal{R}^{N+K}$ is a vector containing all ones.

We note that $-\mathbf{L}$ is a lower triangular matrix having negative diagonal elements and nonnegative off diagonal elements. This means that $-\mathbf{L}$ is an $\mathcal{M}$-matrix as defined in [25]. According to the properties of such matrices, it is possible to assign the positive constants $\delta_1, \delta_2, \ldots, \delta_K, \delta_{K+1}, \ldots, \delta_N, \delta'_1, \ldots, \delta'_K$ so as to make $\mathbf{H} = \mathbf{DL} + \mathbf{L}^T\mathbf{D}$ positive definite. Thus the first term of (5.31) will be a negative number for all nonzero $(\mathbf{e}_1^T, \mathbf{e}_2^T, \mathbf{e}_3^T)$.

Proceeding along the same line as the previous proofs we now complete the square involving $(\tilde{\mathbf{y}}_1^T, \tilde{\mathbf{y}}_2^T)^T$ to get

$$\begin{aligned}\dot{V}_{(5.22)}(\hat{e}, z, \phi) \leq &- (\mathbf{e}_1^T, \mathbf{e}_2^T, \mathbf{e}_3^T)(\mathbf{H} - \frac{\xi^2\bar{\delta}^2}{2\underline{\delta}\rho}\mathbf{M}\mathbf{M}^T)\begin{pmatrix}\mathbf{e}_1\\ \mathbf{e}_2\\ \mathbf{e}_3\end{pmatrix} - 2\underline{\delta}\sigma\left\|\phi + \rho\bar{\theta}\right\|^2\\ &- 2\underline{\delta}\chi\rho\left\|\begin{pmatrix}\tilde{\mathbf{y}}_1\\ \tilde{\mathbf{y}}_2\end{pmatrix} - \frac{\xi\bar{\delta}}{2\underline{\delta}\rho}\mathbf{M}^T\begin{pmatrix}\mathbf{e}_1\\ \mathbf{e}_2\\ \mathbf{e}_3\end{pmatrix}\right\|^2 + 2\bar{\delta}\xi\mathbf{v}^T\begin{pmatrix}\mathbf{e}_1\\ \mathbf{e}_2\\ \mathbf{e}_3\end{pmatrix}\\ &+ 2\bar{\delta}\sigma\left\|\theta^* - \rho\bar{\theta}\right\|^2\\ &\qquad\qquad \forall(\hat{e}, z, \phi) \in \Omega\end{aligned} \tag{5.32}$$

Inequality (5.32) can be written in the more compact form

$$\begin{aligned}\dot{V}_{(5.22)}(\hat{e}, z, \phi) \leq &- \zeta\left\|(\hat{e}, z)\right\|^2 - 2\underline{\delta}\sigma\left\|\phi + \rho\bar{\theta}\right\|^2 + \zeta'\left\|(\hat{e}, z)\right\| + \eta\\ &\forall(\hat{e}, z, \phi) \in \Omega\end{aligned} \tag{5.33}$$

where ζ', and η are positive numbers, and ζ is the minimum eigenvalue of $\mathbf{H} - (2\underline{\delta}\rho)^{-1}\,\xi^2\bar{\delta}^2\,\mathbf{M}\mathbf{M}^T$. Since $\mathbf{H}$ is a positive definite matrix, we can choose ρ sufficiently large that ζ is a positive number. In that case, (5.33) implies

$$\dot{V}_{(5.22)}(\hat{e}, z, \phi) \leq -\mu V(\hat{e}, z, \phi) + \mu' V^{1/2}(\hat{e}, z, \phi) + \eta, \quad \forall(\hat{e}, z, \phi) \in \Omega, \tag{5.34}$$

where

$$\begin{aligned}\mu &\leq \bar{\delta}^{-1}\min\left[\lambda_{\max}^{-1}(\hat{H})\zeta, \lambda_{\max}^{-1}(H')\zeta, 2\lambda_{\min}(\Gamma)\underline{\delta}\sigma\right]\\ \mu' &\geq \underline{\delta}^{-1}\max\left[\lambda_{\min}^{-1}(\hat{H}), \lambda_{\min}^{-1}(H')\right]^{1/2}\zeta'\end{aligned} \tag{5.35}$$

From (5.34) we can conclude that $V(\hat{e}, z, \phi)$ decreases monotonically along any solution of $\hat{\mathbf{S}}_e$ until the solution reaches the compact set

$$\Omega_f = \{(\hat{e}, z, \phi) \in \Omega : V(\hat{e}, z, \phi) \leq V_f\}, \tag{5.36}$$

where

$$V_f = (2\mu)^{-2}\left[\mu' + \left(\mu'^2 + 4\mu\eta\right)^{1/2}\right]^2 \tag{5.37}$$

Therefore, the solutions $(\hat{e}, z, \phi)(t; t_0, \hat{e}_0, \phi_0)$ of $\mathbf{S}_e$ are ultimately bounded with respect to the bound V_f. **Q.E.D.**

(5.38) **REMARK.** Again, if we choose $\sigma \propto \rho^{-3}$ and $\Gamma \propto \rho^3$ and increase ρ, we find that $V_f \sim \rho^{-1}$, as before, and the residual set can be made as small as desired.

(5.39) **EXAMPLE.** We illustrate the adaptive algorithm using the dual pendulum example introduced in Chapter 3. This time we assume that the only measureable variable is

$$y_i = (\,1.0 \quad 0.1\,)\,x_i \quad i = 1, 2 \tag{5.40}$$

and we wish to track the motion of the reference model

$$A_{mi} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}, b_{mi} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, c_{mi}^T = (\,0.5 \quad 0.5\,), \quad i = 1, 2. \tag{5.41}$$

The pre- and post-compensators are given by

$$\begin{aligned} \dot{z}_{pi} &= -z_{pi} + u_i \\ \dot{z}_{fi} &= -z_{fi} + y_i \end{aligned} \qquad i = 1, 2 \tag{5.42}$$

and we apply the reference signal described by (4.24). The results are shown in Figure 5.2. Notice that initially, the pendulums begin to fall, but as the parameters adjust, tracking error approaches the low magnitudes that were observed with the state feedback algorithm.

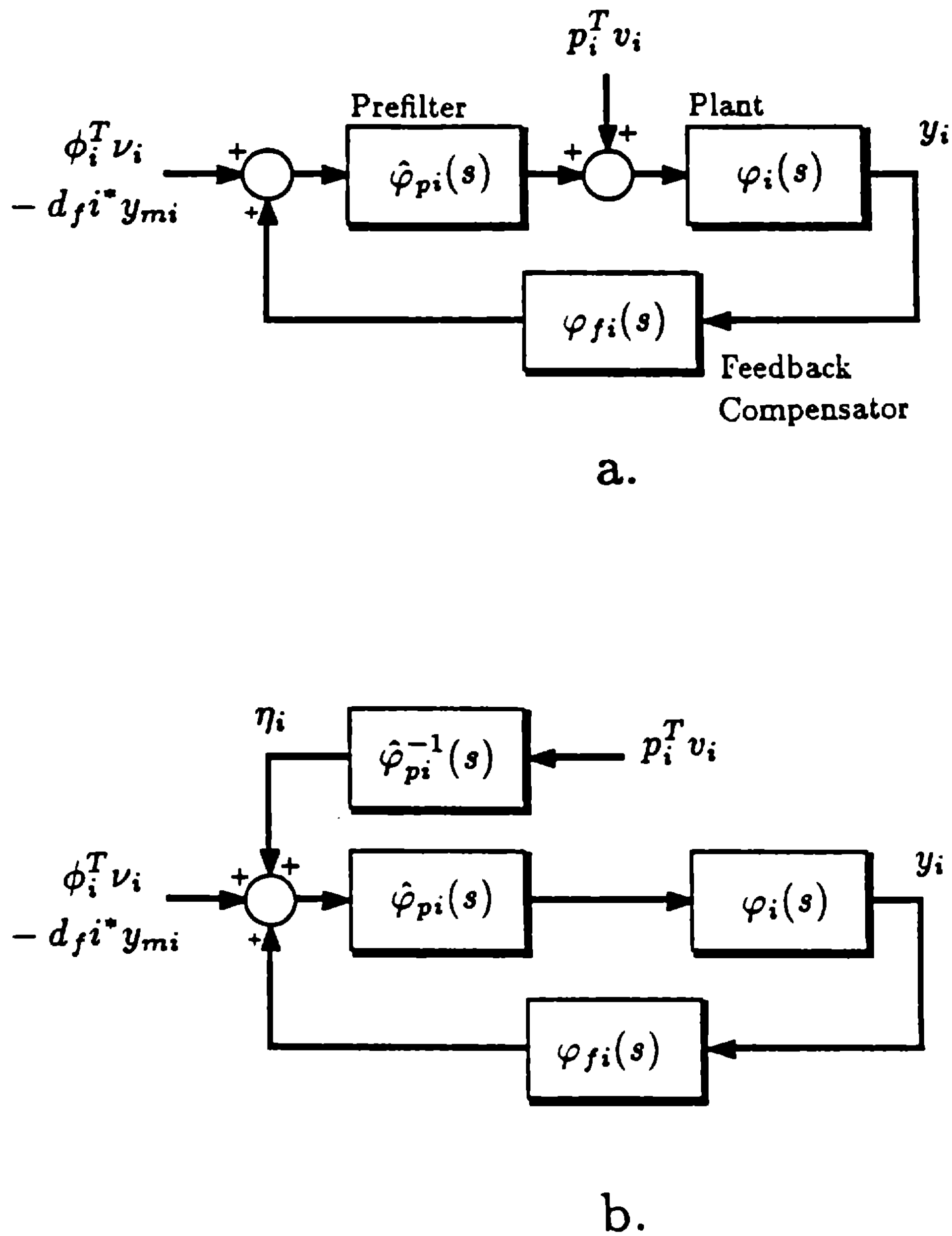

Figure 5.1. Reflecting interconnection disturbances to the input.

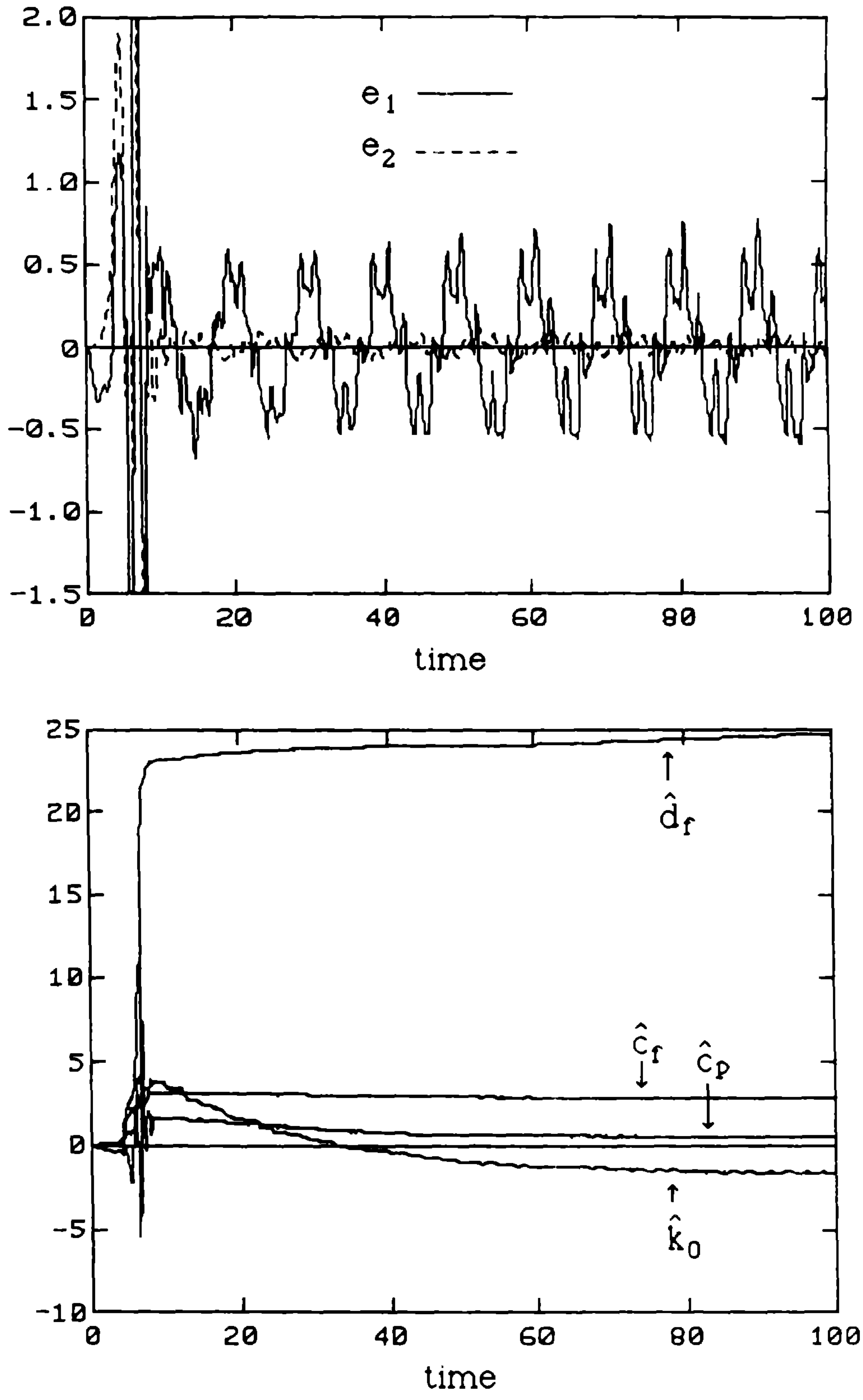

Figure 5.2. Tracking error and gains, output feedback scheme.

6

Application to a Wider Class of Systems

We now examine the wider class of stabilizable decentralized systems studied by Ikeda and Siljak [5] to see if we can apply adaptive control to such systems. The system of interest is described by

$$\mathbf{S}: \quad \dot{x}_i = A_i x_i + b_i u_i + \sum_{j=1}^{N} A_{ij}(x,t)x_j \quad i \in N \tag{6.1}$$

where $A_{ij}(x,t) = [a_{pq}^{ij}(x,t)]$ is an $n_i \times n_j$ matrix of uniformly bounded functions of the arguments x and t. We assume that the pair A_i, b_i are given in the controller canonical form (2.4). Relating (6.1) to the structure introduced in section 2, we have set

$$f_i(w,t) = \sum_{j=1}^{N} h_{ij}(u\,,t)w_j \tag{6.2}$$

and defined

$$A_{ij}(x,t) = P_i h_{ij} Q_j \tag{6.3}$$

This is a slight restriction on the allowable type of interconnection function $f_i(w,t)$, but, in light of the results in [5] we can now replace the range of b condition (3.1) with a weaker condition for structural stabilizability, stated as follows.

Associate with each matrix A_{ij} the integers m_{ij}, where

$$m_{ij} = \begin{cases} \max\limits_{(p,q):\, a_{pq}^{ij} \neq 0} \{q - p\} & A_{ij} \neq 0 \\ -n & A_{ij} = 0 \end{cases} \tag{6.4}$$

where $n = \sum_{i=1}^{N} n_i$. The integers m_{ij} can be interpreted in terms of the structure of A_{ij} as the distance between the main diagonal and a border line of the nonzero elements in A_{ij} which is parallel to the main diagonal. Define a subset I =

$\{i_1, i_2, \ldots, i_r\}$ of the index set N for which $i_1 < i_2 < \ldots < i_r$, $r \leq N$. By permuting the elements of I we form a set $J = \{j_1, j_2, \ldots j_r\}$, also a subset of N. The condition for stabilizability is that the inequality

$$\sum_{k=1}^{r}(m_{i_k j_k} - 1) < 0 \tag{6.5}$$

holds for all subsets I and J of N.

We now proceed to design an adaptive controller to stabilize systems meeting criterion (6.5). The adaptive control laws are similar in form to those used earlier. Control is given by

$$u_i = \theta_i^T \nu_i \tag{6.6}$$

where

$$\nu_i = \begin{pmatrix} x_i \\ r_i \end{pmatrix}, \quad \theta_i = \begin{pmatrix} k_i \\ k_{0i} \end{pmatrix} \tag{6.7}$$

and adaptation is given by

$$\dot{\theta}_i = -\Gamma_i(\bar{k}_i^T e_i)\nu_i - \Gamma_i \sigma_i \theta_i \tag{6.8}$$

where $e_i \in \mathcal{R}^{n_i}$: $e_i = x_i - x_{mi}$, x_{mi} is the reference model state from equation (2.5), and $\bar{k}_i \in \mathcal{R}^{n_i}$ is defined below.

Substituting (6.6) into (6.1) gives the closed loop system

$$\hat{\mathbf{S}}_i: \quad \dot{x}_i = A_{mi}x_i + b_{mi}r_i + b_i\phi_i^T\nu_i + \sum_{j=1}^{N} A_{ij}(x,t)x_j \quad i \in N. \tag{6.9}$$

Subtracting the equation for the reference model, $\mathbf{M}_i$ (2.5) we get the error differential equation

$$\hat{\mathbf{S}}_{ei}: \quad \dot{e}_i = A_{mi}e_i + b_i\phi_i^T\nu_i + \sum_{j=1}^{N} A_{ij}(x,t)(e_j + x_{mj}(t)) \quad i \in N. \tag{6.10}$$

The reference models must be chosen so that each A_{mi} has all of its eigenvalues placed distinctly on the negative real axis at $\{-\alpha\lambda_{1i}, -\alpha\lambda_{2i}, \ldots, -\alpha\lambda_{n_i i}\}$

where $\lambda_{ki} > 0$ for $k \in \{1, 2, \ldots, n_i\}$ and $i \in N$. The constant α is a design parameter to be assigned later. Because the eigenvalues of A_{mi} are at distinct locations, we can apply a nonsingular transformation

$$\tilde{e}_i = T_i^{-1} e_i \tag{6.11}$$

to $\hat{\mathbf{S}}_e$ and get the equation of the transformed system

$$\hat{\mathbf{S}}_{ei}: \quad \dot{\tilde{e}}_i = \Lambda_{mi}\tilde{e}_i + \tilde{b}_i \phi_i^T \nu_i + \sum_{j=1}^{N} \tilde{A}_{ij}(x,t)\,(\tilde{e}_j + \tilde{x}_{mj}(t)) \tag{6.12}$$

where

$$\begin{aligned} \Lambda_{mi} &= T_i^{-1} A_{mi} T_i = \mathrm{diag}\{-\alpha\lambda_{1i} \ -\alpha\lambda_{2i}, \ldots, -\alpha\lambda_{n,i}\} \\ \tilde{A}_{ij} &= T_i^{-1} A_{ij} T_j \\ \tilde{b}_i &= T_i^{-1} b_i \\ \tilde{x}_{mi}(t) &= T_i^{-1} x_{mi}(t) \end{aligned} \quad . \tag{6.13}$$

The transformation matrix T is given by

$$T = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ -\alpha\lambda_{1i} & -\alpha\lambda_{2i} & \cdots & -\alpha\lambda_{n,i} \\ -(\alpha\lambda_{1i})^2 & -(\alpha\lambda_{2i})^2 & \cdots & -(\alpha\lambda_{n,i})^2 \\ \vdots & \vdots & \ddots & \vdots \\ -(\alpha\lambda_{1i})^{n_i-1} & -(\alpha\lambda_{2i})^{n_i-1} & \cdots & -(\alpha\lambda_{n,i})^{n_i-1} \end{pmatrix}. \tag{6.14}$$

We now define

$$\bar{k}_i = T_i^{-T} T_i^{-1} b_{mi}, \tag{6.15}$$

and the local adaptation law (6.8) can be rewritten as

$$\dot{\theta}_i = -\Gamma_i(\tilde{b}_i^T \tilde{e}_i)\nu_i - \Gamma_i \sigma_i \theta_i . \tag{6.15}$$

(6.16) **Theorem**. There exists a suffieiently large number $\alpha > 1$ such the motion of the system $\hat{\mathbf{S}}_e$ is globally ultimately bounded with respect to $\tilde{e}$, and $\theta(t)$ is uniformly ultimately bounded.

Proof. The proof is similar to that given by Ikeda and Siljak [5] with the necessary modifications for the adaptive part. We use the function

$$V(\tilde{e},\phi)=\sum_{i=1}^{N}\delta_i\left[k_{0i}^{*}\tilde{e}_i^{T}\tilde{e}_i+\phi_i^{T}\Gamma^{-1}\phi_i\right] \tag{6.17}$$

and take the total time derivative with respect to (6.12) to get

$$\begin{aligned}\dot{V}(\tilde{e},\phi)_{(6.12)}=\sum_{i=1}^{N}\delta_i\Bigg[&2k_{0i}^{*}\tilde{e}_i^{T}\Lambda_i\tilde{e}_i+2\tilde{e}_i^{T}\sum_{j=1}^{N}\tilde{A}_{ij}(x,t)\tilde{e}_j\\&+2\tilde{e}_i^{T}\sum_{j=1}^{N}T_i^{-1}A_{ij}(x,t)x_{mj}(t)-2\sigma\phi_i^{T}\phi_i-2\sigma\phi^{T}\theta_i^{*}\Bigg].\end{aligned} \tag{6.18}$$

Taking norms of terms on the right hand side of (6.18) we have the inequality

$$\begin{aligned}\dot{V}_{(6.12)}\le\sum_{i=1}^{N}\delta_i\Bigg[&-2\alpha k_{0i}^{*}\lambda_i\|\tilde{e}_i\|^2+2\|\tilde{e}_i\|\sum_{j=1}^{N}\|\tilde{A}_{ij}\|\|\tilde{e}_j\|\\&+2\|\tilde{e}\|\sum_{j=1}^{N}\|T^{-1}\|\|A_{ij}\|\sup_t\|x_{mj}(t)\|\\&-\sigma\|\phi_i\|^2+\sigma\|\theta_i^{*}\|^2\Bigg]\end{aligned} \tag{6.19}$$

where $\lambda_i=\min_k(\lambda_{ik})$. In (6.19) we have used the shorthand notation $\|A_{ij}\|$ to mean $\sup_{x,t}\|A_{ij}(x,t)\|$. The supremum exists because we have assumed that $A_{ij}(x,t)$ is a uniformly bounded function of its arguments.

To compute $\|\tilde{A}_{ij}\|$, we note that the transformation matrix T_i in (6.14) can be factored as

$$T_i=R_i\hat{T}_i \tag{6.20}$$

where

$$\hat{T}_i=\begin{pmatrix}1&1&\cdots&1\\-\lambda_{1i}&-\lambda_{2i}&\cdots&-\lambda_{n_ii}\\-\lambda_{1i}^2&-\lambda_{2i}^2&\cdots&-\lambda_{n_ii}^2\\\vdots&\vdots&\ddots&\vdots\\-\lambda_{1i}^{n_i-1}&-\lambda_{2i}^{n_i-1}&\cdots&-\lambda_{n_ii}^{n_i-1}\end{pmatrix} \tag{6.20}$$

and

$$R_i = \mathrm{diag}\{1, \alpha, \alpha^2, \ldots, \alpha^{n_i - 1}\}. \tag{6.21}$$

Then

$$\begin{aligned} \|\tilde{A}_{ij}\| &= \|\hat{T}_i^{-1} R_i^{-1} A_{ij} R_j \hat{T}_j\| \\ &\leq \|\hat{T}_i^{-1}\| \|\hat{T}_j\| \|R_i^{-1} A_{ij} R_j\|. \end{aligned} \tag{6.22}$$

Using (6.4), we see that the p, q element of $R_i^{-1} A_{ij} R_j$ is no greater than

$$\alpha^{m_{ij}} |a_{pq}^{ij}|.$$

Therefore

$$\|R_i^{-1} A_{ij} R_j\| \leq \alpha^{m_{ij}} \sum_{p=1}^{n_i} \sum_{q=1}^{n_j} |a_{pq}^{ij}|. \tag{6.23}$$

We can therefore conclude that

$$\|\tilde{A}_{ij}\| \leq \alpha^{m_{ij}} \xi_{ij} \tag{6.24}$$

where ξ_{ij} is a constant, independent of α:

$$\xi_{ij} = \|\hat{T}_i^{-1}\| \|\hat{T}_j\| \sum_{p=1}^{n_i} \sum_{q=1}^{n_j} |a_{pq}^{ij}|. \tag{6.25}$$

Also, we see that

$$\|T_i^{-1}\| \leq \|\hat{T}_i^{-1}\| \|R_i^{-1}\| = \|\hat{T}_i^{-1}\|, \tag{6.26}$$

and substituting (6.25) and (6.26) into (6.19), we get

$$\begin{aligned} \dot{V}_{(6.12)} \leq \sum_{i=1}^{N} \delta_i \Bigg[&- 2\alpha k_{0i}^* \lambda_i \|\tilde{e}_i\|^2 + 2\alpha \|\tilde{e}_i\| \sum_{j=1}^{N} \xi_{ij} \alpha^{m_{ij}-1} \|\tilde{e}_j\| \\ &+ 2\|\tilde{e}_i\| \sum_{j=1}^{N} \|\hat{T}_i^{-1}\| \|A_{ij}\| \sup_t \|x_{mj}(t)\| - \sigma \|\phi_i\|^2 + \sigma \|\theta_i^*\|^2 \Bigg]. \end{aligned} \tag{6.27}$$

We now define the $N \times N$ matrix $W = [w_{ij}]$, whose elements are given by

$$w_{ij} = \begin{cases} k_{0i}^* \lambda_i - \xi_{ij} \alpha^{m_{ii}-1} & i = j \\ -\xi_{ij} \alpha^{m_{ij}-1} & i \neq j \end{cases} \tag{6.28}$$

and define $D = \mathrm{diag}\{\delta_1, \delta_2, \ldots, \delta_N\}$. We can now rewrite inequality (6.27) as

$$\begin{aligned} \dot{V}_{(6.12)} \leq & -\alpha\lambda_{\min}(DW + W^T D)\|\tilde{e}\|^2 \\ & + 2\chi\|\tilde{e}\| - \sigma\underline{\delta}\|\phi\|^2 + \sigma\bar{\delta}\|\theta^*\|^2 \end{aligned} \tag{6.29}$$

where $\chi = \max_{i,j}\{\|\delta_i \hat{T}_i^{-1}\| \; \|A_{ij}\| \sup_t \|x_{mj}(t)\|\}$, $\underline{\delta} = \min_i \delta_i$, and $\bar{\delta} = \max_i \delta_i$. From the result of Lemma A.13 in [5], it is possible to choose a sufficiently large α such that the leading principal minors of W are all positive, thereby making W an **M** matrix as defined in [25]. According to the properties of such a matrix, positive constants δ_i, $i \in N$ can be selected to make the matrix $DW + W^T D$ positive definite, and its minimum eigenvalue will be a positive number.

The inequality can then be simplified to

$$\dot{V}_{(6.12)} \leq -\mu V + \mu' V^{1/2} + \eta \tag{6.30}$$

where

$$\begin{aligned} \mu \leq & \bar{\delta}^{-1} \min\left[\bar{k}_0^{*-1}\alpha\lambda_{\min}(DW + W^T D), \lambda_{\min}(\Gamma)\sigma\right] \\ \mu' \geq & 2\chi\underline{\delta}^{-1}\underline{k}_0^{*-1} \\ \eta = & \sigma\|\phi\|^2 \end{aligned} \tag{6.31}$$

$\bar{k}_0^* = \max_i\{k_{0i}^*\}$, $\underline{k}_0^* = \min_i\{k_{0i}^*\}$, and $\Gamma = \mathrm{diag}\{\Gamma_1, \Gamma_2, \ldots, \Gamma_N\}$. The differential inequality (6.30) implies that $V(e, \phi)$ decreases monotonically along any solution of $\hat{\mathbf{S}}_e$ until the solution reaches the compact set

$$\Omega_f = \{(e, \phi) \in \mathcal{R}^n \times \mathcal{R}^{n+N} : V(e, \phi) \leq V_f\} \tag{6.32}$$

where

$$V_f = (2\mu)^{-2}\left[\mu' + \left(\mu'^2 + 4\mu\eta\right)^{1/2}\right]^2 \tag{6.33}$$

Therefore, the solutions $(\hat{e}, \phi)(t; t_0, \hat{e}_0, \phi_0)$ of $\hat{\mathbf{S}}_e$ are ultimately bounded with respect to the bound V_f. **Q.E.D.**

(6.34) **Remark**. The essential condition for stability here is that α must be chosen large enough to make W a $\mathcal{M}$ matrix, that is, the designer must choose sufficiently stable reference models in order to overcome interconnection disturbances. This is a stronger requirement than before, where any stable reference model would guarantee stability. The benefit gained from having enough information to choose α is that a much broader class of interconnection structures in the plant can now be tolerated.

(6.35) **Remark**. Earlier results in decentralized adaptive control [22] require that W be a $\mathcal{M}$ matrix, however no design technique is given to assure that W meets this condition. The proof above suggests that by selecting larger and larger α until W meets the $\mathcal{M}$ matrix condition we can design a stable adaptive control law. The proof guarantees that a sufficiently large, but finite, α exists. Furthermore, in view of (6.30) through (6.33), we see that by choosing larger α and Γ, the residual set can be made as small as desired.

7
Application to Robotics

In this chapter we examine the use of decentralized adaptive schemes for robot arm joint motion control. The control of robotic manipulators is particularly challenging because of the highly nonlinear and trajectory dependent dynamics coupled with the need for fast on-line implementation of the control laws. A decentralized control design, with one controller dedicated for each joint, is an attractive approach since such designs can be made robust to the nonlinearities and they can be implemented with parallel processors for the necessary speed advantage. Adaptation provides additional robustness to the parametric variations due to a changing load or arm configuration.

The development of high performance robot arm controllers has been rapid over the past several years. The earliest designs, known as independent joint control, use a servo motor and a conventional PID controller at each joint [56]. Such controllers are limited in performance unless the disturbance torques which interconnect the single joint subsystems are compensated for [34]. In a more sophisticated design, known as the computed torque technique, the disturbance torques are explicitly canceled at each joint [35,36]. This method, however, requires a complex, nonlinear full-state-feedback control law and is therefore costly in terms of computer resources. Also, it requires that the inertial characteristics of the arm and its load be known at all times.

To compenstate for coupling torques, several schemes for enhancing independent joint control have been proposed. These include the variable structure methods [37–40], the acceleration estimation and feedback schemes [41–43], and the adaptive control schemes [28,29,44–48] A survey of the adaptive robot control methods is given in [49].

The variable structure control methods [38–40] are robust, decentralized, and simple to implement, (this type of control law is often categorized as model-referenced adaptive control, apparently because hyperstability arguments are used in the stability proofs) however, they depend on a high frequency switching control law, which unfortunately could excite parasitic high frequency modes in a robot arm. This is to be contrasted with the relatively smooth control, with slowly varying linear feedback gains, proposed in this thesis.

Acceleration feedback schemes have shown promise in experimental work [43]. This approach, however, requires either taking numerical derivatives of discrete joint position measurements, or using additional transducers to measure acceleration directly.

Adaptive independent joint control was first proposed in [44] where, in the the stability analysis, the effect of the coupling torques was ignored. Since then, several papers have appeared in the literature which propose adaptive decentralized algorithms (for example, [45], [47], and [28]). For convience in designing the adaptation rules, only simplified models for the interconnection torques are hypothesized. These simplifications, unfortunately, are not truely representative of the robot arm's nonlinear dynamics. For this reason, asymptotic stability of the adaptive control laws is left in question, although the simulation and experimental results look encouraging.

In later research on *centralized* adaptive robot control, the full nonlinear dynamics are taken into account, and asymptotically stable algorithms have been developed [50–52]. While these control laws give the desired robustness to parametric uncertainties, they are based on the computed torque technique, thus the decentralized structure of the controller is sacrificed.

To design a stable decentralized robot controller, the nonlinear coupling terms must be properly taken into account according to the principls of nonlinear large scale system theory. Vucobratović and Stokić [53,54] use a large scale systems approach to show the stabilty and robustness of a (nonadaptive) decentralized law. Their proposed scheme uses a combination of global and local control where the global controller uses computed torque to aid in decoupling the subsystems, while the local controller provides robustness to perturbations. In [46] an adaptation mechanism is added on for payload identification.

In this chapter, we present an adaptive decentralized control law for robot joint control, and prove that it has local stability properties. The important distinctions from earlier results are 1) the control law is completely decentralized with no global controller necessary, and 2) the stability analysis takes into account all of the nonlinear coupling terms. It is also important to point out that the proposed controller does not require acceleration feedback nor does it use high frequency gain switching.

An important property of robot arm dynamics is that the dynamic interaction torques are within the range space of the control (that is, the conditions (3.1) are satisfied), therefore, in view of the results of the earlier chapters, we should expect that local high gain control is sufficient to stabilize the overall system. What distinguishes robot arms from our earlier systems of interest however, is the fact that the robot arm system is not *input-decentralized*. That is, when the equations of motion are written in the state space form (2.3), the B matrix is full, not block diagonal as has been our assumption up to now This means that the control torque intended for one joint cross couples to the other joints, and the decentrally controlled subsystems are thus strongly coupled in a static manner. Fixed, high gain control laws for decentralized systems with cross-connected inputs are given in [55]. We extend on that idea, and exploit the special properties of the robot arm to show that our decentralized adaptive law is stable.

The dynamics of a N joint robot manipulator can be expressed as [56]

$$J(q, l)\ddot{q} + V\dot{q} + c(q, \dot{q}, l) + g(q, l) = \tau \tag{7.1}$$

where

$$
\begin{aligned}
q &= N \text{ by 1 vector of joint positions} \\
l &= \text{a parameter vector describing mass and} \\
&\qquad \text{orientation of payload} \\
J(q,\, l) &= N \text{ by } N \text{ inertia matrix} \\
V &= N \text{ by } N \text{ diagonal viscous friction matrix} \\
c(q,\, \dot{q},\, l) &= N \text{ by 1 vector defining Coriolis and} \\
&\qquad \text{centrifugal terms} \\
g(q,\, l) &= N \text{ by 1 vector defining the gravity terms,} \\
&\qquad \text{and} \\
\tau &= N \text{ by 1 vector of input torques}
\end{aligned}
$$

The parameter vector l may be time-varying, but it is uniformly bounded, reflecting the fact that the arm is designed to handle loads up to a maximum size.

The moment of inertia matrix $J(q,l)$ is always positive definite regardless of arm configuration or load, therefore its inverse $M(q,l) = [J(q,l)]^{-1}$ exists and is also positive definite.

The i'th component of Coriolis force has the form

$$
c_i(q, \dot{q}, l) = \sum_{j=1}^{N} \sum_{k=1}^{N} c_{ijk}^0(q, l)\dot{q}_j \dot{q}_k, \quad i \in \boldsymbol{N} \tag{7.2}
$$

where $c_{ijk}^0(q; l)$ are uniformly bounded functions of q and l.

The gravity force on the i'th joint can be expanded in a power series in q_i about the nominal arm configuration $q = q_0$ and load $l = l_0$ as

$$
g_i(q, l) = g_i^0 + g_i' q_i + \tilde{g}_i(q, l) \quad i \in \boldsymbol{N} \tag{7.3}
$$

where

$$
\begin{aligned}
g_i^0 &= g_i(q_0, l_0) \\
g_i' &= \left.\frac{\partial g_i}{\partial q_i}\right|_{\substack{q=q_0 \\ l=l_0}} \\
\tilde{g}_i(q,l) &= g_i(q,l) - g_i' q_i - g_i^0 .
\end{aligned}
\tag{7.4}
$$

We wish to express equation (7.1) in the familiar first order differential equation form. Defining $r = \dot{q}$ we have

$$
\begin{aligned}
\dot{q} &= r \\
\dot{r} &= M(q,l)\left[-Vr - c(q,r,l) - g(q,l) + \tau\right] .
\end{aligned}
\tag{7.5}
$$

The positive definite matrix $M(q,l)$ can be decomposed as

$$
M(q,l) = M_D - M_C(q,l) \tag{7.6}
$$

where $M_D = \mathrm{diag}\{m_1, m_2, \ldots, m_N\}$, $m_i > 0$, $i = 1, 2, \ldots, N$ and $M_C(q,l)$ is a positive semi-definite matrix for all q and l. Thus

$$
\begin{aligned}
\dot{r} &= \left[M_D + M_C(q,l)\right]\left[-Vr - c(q,r\ l) - g(q,l) + \tau\right] \\
&= - M_D V r - M_D c(q,r,l) - M_D g(q,l) + M_D \tau \\
&\quad + M_C(q,l)\left[-Vr - c(q\ r,l) - g(q,l) + \tau\right] .
\end{aligned}
\tag{7.7}
$$

Substituting (7.3) and writing the equation explicitly for each joint we have

$$
\begin{aligned}
\dot{q}_i &= r_i \\
\dot{r}_i &= - m_i g_i' q_i - m_i v_i r_i + m_i \tau_i \\
&\quad + m_i d_i + m_i f_i(q,r,l) + \sum_{j=1}^{N} m_{ij}(q,l)\tau_j \\
&\qquad i \in \boldsymbol{N}
\end{aligned}
\tag{7.8}
$$

where v_i is the i'th diagonal element of V, m_{ij} is the i,j element of M_C and

$$
\begin{aligned}
d_i &= - g_i^0 - m_i^{-1} \sum_{j=1}^{N} m_{ij}^0 g_j^0 \\
f_i(q,r,l) &= - \left[c_i(q,r,l) + \tilde{g}_i(q,l)\right] \\
&\quad - m_i^{-1} \sum_{j=1}^{N} \left\{\tilde{m}_{ij}(q,l) g_j^0 + m_{ij}(q,l)\left[v_j r_j + c_j(q,r,l) + g_j' q_j + \tilde{g}_j(q,l)\right]\right\} \\
&\qquad i \in \boldsymbol{N}
\end{aligned}
\tag{7.9}
$$

where $m_{ij}^0 = m_{ij}(q_0, l_0)$ and $\tilde{m}_{ij}(q,l) = m_{ij}(q,l) - m_{ij}^0$. In (7.8) d_i represents the constant gravity disturbance at the nominal configuration and $f_i(q,r,l)$ represents homogeneous interconnection forces, that is $f_i(q_0, 0, l_0) = 0$ for $i \in \boldsymbol{N}$.

To attain a system representation similar to that studied in Chapter 3, we define the subsystem state vectors $x_i = (q_i - q_{0i}, r_i)^T$, the overall system state vector $x = (x_1^T, x_2^T, \ldots, x_N^T)^T \in \mathcal{R}^n$, the local control variables $u_i = \tau_i$, and overall control vector $u = (u_1, u_2, \ldots, u_N)^T$. Also, we now use the notation $f_i(t,x)$ to mean $f_i(q,r,l)$ where $t \in \mathcal{R}^+$ denotes the independent time variable. This reflects the use of the state vector x in place of q, r, and the fact that the load l may be time-varying. The function $f_i(t,x) : \mathcal{R}^+ \times \mathcal{R}^{2N} \to \mathcal{R}$ is continuous and homogeneous in x and piecewise continuous in t. Also, we now use $b_{ij}(t,x)$ to denote $m_i^{-1} m_{ij}(q,l)$. The function $b_{ij}(t,x) : \mathcal{R}^+ \times \mathcal{R}^{2N} \to \mathcal{R}$ is uniformly bounded and continuous in both of its arguments. We can now rewrite (7.8) as

$$\mathbf{S}_i : \dot{x}_i = A_i x_i + b_i u_i + b_i \left[f_i(t,x) + \sum_{j=1}^{N} b_{ij}(t,x) u_j + d_i \right] \quad i \in \boldsymbol{N} \tag{7.10}$$

where

$$A_i = \begin{pmatrix} 0 & 1 \\ -m_i g_i' & -m_i v_i \end{pmatrix}, \quad b_i = \begin{pmatrix} 0 \\ m_i \end{pmatrix}. \tag{7.11}$$

For a robot arm, the functions $f_i(\cdot,\cdot)$, $b_{ij}(\cdot,\cdot)$ are sufficiently smooth to assure unique solutions $x(t; t_0, x(t_0))$ to (7.10) for piecewise continuous inputs $u_i(t)$. Also there exists a unique equilibrium point at $x = 0$ when $u_i = g_i^0$, $\forall i \in \boldsymbol{N}$.

Comparing (7.10) to (2.1) and (3.1) we see that the robot system is similar to the system treated in Chapter 3, however, two additional disturbance terms appear. These additional terms represent the coupling of control torques among the joints and the constant disturbance due to gravity.

(7.12)**Remark**. Dynamic interactions among the single joint subsystems are caused by the nonlinear functions $f_i(t,x)$. These interaction disturbances enter the subsystems through the range space of b_i. However, unlike before, the interconnection function, $f_i(t,x)$ is not conically bounded as in (2.6). In view of (7.2) and (7.9) we see that Coriolis and centrifugal forces cause the interconnection torques to behave as *quadratic* functions of joint velocity r. Thus we have the relation

$$\|f_i(t,x)\| \le \sum_{j=1}^{N} \alpha_{ij}\|x_j\|^2 + \beta_{ij}\|x_j\| \qquad \forall (t,x) \in \mathcal{R} \times \mathcal{R}^{2N}, \quad i \in N \tag{7.13}$$

(7.14)**Remark**. Also unlike before, control signals interact in a time-dependent manner through the interaction matrix $B_C(t,x) := [b_{ij}(t,x)] = M_D^{-1} M_C(q,l)$. Two important facts about this interaction, characteristic of robot arms and other mechanical systems of the type (7.1), should be noted. First of all, the interaction of controls is through the range space of b_i. Secondly, M_D is a diagonal matrix with positive elements and $M_C(q,l)$ is positive semi-definite, thus the interaction matrix B_C is positive semi-definite. We will exploit both these conditions in the proof of stability.

The control objective is to follow the reference model

$$\mathbf{M}_i : \dot{x}_{mi} = A_{mi}x_{mi} + b_{mi}r_{mi} \quad i \in N \tag{7.15}$$

where

$$A_{mi} = A_i + b_i k_i^{*T}, \quad b_{mi} = b_i k_{0i}^* \tag{7.16}$$

and k_i^* and k_{0i}^* are the model-matching gain settings.

The proposed decentralized adaptive control law is

$$u_i = \theta_i^T \nu_i \quad i \in N \tag{7.17}$$

where

$$\theta_i(t) = (k_{0i}(t), k_i^T(t), k_{Ii}(t))^T \in \mathcal{R}^4 \tag{7.18}$$

are the adapted, time-varying, estimates of the model-matching gains, and

$$\nu_i(t) = (r_i, e_i^T, 1)^T \in \mathcal{R}^4 \tag{7.19}$$

are the regressor signals, where $e_i = x_i - x_{mi}$. Note that we have added an additional signal $k_{Ii}(t)$ in the feedback law. This signal will act as an integral feedback term to cancel the constant disturbance d_i. The matching setting of $k_{Ii}(t)$ is $k_{Ii}^* = -d_i$.

The adaptation law is then

$$\dot{\theta}_i = -\Gamma_i(\bar{k}_i^T e_i)\nu_i - \sigma\Gamma_i(\theta_i + \rho\bar{\theta}_i) \quad i \in N \tag{7.20}$$

where $\bar{k}_i$ is defined by (3.3), (3.4), $\bar{\theta}_i = (0, \bar{k}_i^T, 0)^T$, Γ_i is a 4×4 positive definite matrix, and σ and ρ are positive design constants.

Defining $\theta^* = (k_{0i}^*, k_i^{T*}, k_{Ii}^*)^T$ and $\phi = \theta - \theta^*$ and subtracting (7.15) from (7.10) we have the error differential equations

$$\begin{aligned}\hat{\mathbf{S}}_{ei} : \dot{e}_i =& A_{mi}e_i + b_i\big[\phi_i^T\nu_i - k_i^{*T}x_{mi} \\ &+ f_i(t,x) + \sum_{j=1}^{N} b_{ij}(t,x)(\theta_j^{*T}\nu_j + \phi_j^T\nu_j)\big] \quad i \in N \\ \dot{\phi}_i =& -\sigma\Gamma_i\phi_i - \sigma\Gamma_i\theta_i^* - \sigma\Gamma_i\rho\bar{\theta}_i - \Gamma_i(\bar{k}_i^T e_i)\nu_i\end{aligned} \tag{7.21}$$

We denote $(e, \phi) \in \mathcal{R}^{2N} \times \mathcal{R}^{4N}$ as the state of $\hat{\mathbf{S}}_e$.

(7.22) **Theorem**. There exists an attraction set $\Omega \subset \mathcal{R}^{2N} \times \mathcal{R}^{4N}$ such that if the initial condition (e_0, ϕ_0) is within Ω, then the solutions $(e, \phi)(t; t_0, e_0, \phi_0)$ of $\hat{\mathbf{S}}_e$ are ultimately bounded.

Proof. Define the positive function

$$V(e, \phi) = \sum_{i=1}^{N} k_{0i}^* e_i^T H_i e_i + (\phi_i + \rho\bar{\theta}_i)^T\Gamma_i^{-1}(\phi_i + \rho\bar{\theta}_i) \tag{7.23}$$

where H_i is the positive definite matrix that satisfies (3.3). We specify a domain of attraction as $\Omega = \{(e, \phi) : V(e, \phi) \leq V_0\}$ where V_0 is a positive number. By this definition, Ω is a compact set, therefore, for all $(e, \phi) \in \Omega$

$$\begin{aligned}
\|\phi_i + \rho\bar{\theta}_i\| &\leq \bar{\phi}_i \\
\|f_i(t, x)\| &\leq \sum_{j=1}^{N} \xi_{ij}(\|e_i\| + \|x_{mi}\|) \\
\|b_{ij}(t, x)\| &\leq \bar{b}_{ij} \\
& \qquad i \in \mathbf{N}, \quad (e, \phi) \in \Omega
\end{aligned} \tag{7.24}$$

where

$$\begin{aligned}
\bar{\phi} &= \sup_{\Omega} \|\phi + \rho\bar{\theta}\| = V_0^{1/2}\lambda_{\max}^{1/2}(\Gamma) \\
\xi_{ij} &= \alpha_{ij}(\bar{e} + \sup_{\mathcal{R}^+} \|x_{mi}(t)\|) + \beta_{ij} \\
\bar{e} &= \sup_{\Omega} \|e\| = V_0^{1/2}\lambda_{\min}^{-1/2}(H) \\
\bar{b}_{ij} &= \sup_{\mathcal{R}^+ \times \Omega} \|b_{ij}(t, x)\|
\end{aligned} \tag{7.25}$$

and $H = \text{diag}\{k_{01}^* H_1, k_{02}^* H_2, \ldots, k_{0N}^* H_N\}$, $\Gamma = \text{diag}\{\Gamma_1, \Gamma_2, \ldots, \Gamma_N\}$, $e = (e_1^T, e_2^T, \ldots, e_N^T)^T$, $\phi = (\phi_1^T, \phi_2^T, \ldots, \phi_N^T)^T$, and $\bar{\theta} = (\bar{\theta}_1^T, \bar{\theta}_2^T, \ldots, \bar{\theta}_N^T)^T$.

We now take the total time derivative of $V(e, \theta)$ along solutions of $\hat{\mathbf{S}}_e$ to get

$$\begin{aligned}
\dot{V}(e, \phi)_{(7.21)} = \sum_{i=1}^{N} \Big\{ & - k_{0i}^* e_i^T G_i e_i - 2e_i^T \bar{k}_i k_i^{*T} x_{mi} + 2e_i^T \bar{k}_i f_i(t, x) \\
& + 2e_i^T \bar{k}_i \sum_{j=1}^{N} b_{ij}(t, x)\big[\theta_j^{*T}\nu_j + (\phi_j + \rho\bar{\theta}_j)^T \nu_j - \rho\bar{\theta}_j^T \nu_j\big] - 2\rho e_i^T \bar{k}_i \bar{k}_i^T e_i \\
& - 2(\phi_i + \rho\bar{\theta}_i)^T \sigma(\phi_i + \rho\bar{\theta}_i) - 2(\phi_i + \rho\bar{\theta}_i)^T \sigma\theta_i^* \Big\} \\
& \forall (t, e, \phi) \in \mathcal{R} \times \Omega.
\end{aligned} \tag{7.26}$$

Using the block notation introduced in Chapter 3,

$$\begin{aligned}
\dot{V}(e, \phi)_{(7.21)} = & -e^T G e - 2e^T \bar{K}^T K^* x_m + 2e^T \bar{K}^T f(t, x) \\
& + 2e^T \bar{K}^T B_C(\theta^* + \phi + \rho\bar{\theta})^T \nu \\
& - 2\rho e^T \bar{K}^T B_C \bar{\theta}^T \nu - 2\rho e^T \bar{K}^T \bar{K} e \\
& - 2\sigma(\phi + \rho\bar{\theta})^T(\phi + \rho\bar{\theta}) - 2\sigma(\phi + \rho\bar{\theta})^T \theta^* \\
& \forall (t, e, \phi) \in \mathcal{R} \times \Omega.
\end{aligned} \tag{7.27}$$

Note, from (7.25) that

$$||\theta^* + \phi + \rho\bar{\theta}|| \leq ||\theta^*|| + \bar{\phi} \tag{7.28}$$

and

$$||\nu|| \leq (||r|| + ||e|| + N), \tag{7.29}$$

and also, using the definition of $\bar{\theta}$,

$$\bar{\theta}^T \nu = \bar{K}e. \tag{7.30}$$

By completing the square, we also have

$$\begin{aligned} &- 2\sigma(\phi + \rho\bar{\theta})^T(\phi + \rho\bar{\theta}) - 2\sigma(\phi + \rho\bar{\theta})^T\theta^* = \\ &\qquad - \sigma||\phi + \rho\bar{\theta}||^2 - \sigma||\phi + \rho\bar{\theta} + \theta^*||^2 + \sigma||\theta^*||^2. \end{aligned} \tag{7.31}$$

Now, substituting the above into (7.27) and taking norms we get

$$\begin{aligned} \dot{V}(e,\phi)_{(7.21)} = &-e^T Ge + 2||\bar{K}e||||K^* x_m|| + 2||\bar{K}e||||f|| \\ &+ 2||\bar{K}e||||B_C||(||\theta^*|| + \bar{\phi})(||r|| + ||e|| + N) \\ &- 2\rho e^T \bar{K}^T B_C \bar{K}e - 2\rho||\bar{K}e||^2 \\ &- \sigma||\phi + \rho\bar{\theta}||^2 + \sigma||\theta^*||^2 \\ &\qquad \forall (t, e, \phi) \in \mathcal{R} \times \Omega. \end{aligned} \tag{7.32}$$

Since B_C is positive semi-definite (see Remark (7.14)), the term $-2\rho e^T \bar{K}^T B_C \bar{K}e$ can be dropped from the right hand side of the inequality, since it is never positive. Now, using (7.24) $||f(t,x)|| \leq \xi(||e|| + ||x_m||)$. and $||B_C|| \leq \bar{b}$ for some positive constants ξ and $\bar{b}$, so we have

$$\begin{aligned} \dot{V}(e,\phi)_{(7.21)} \leq &- e^T Ge - 2\rho||\bar{K}e||^2 + 2||\bar{K}e||||K^* x_m|| \\ &+ 2\xi||\bar{K}e||(||e|| + ||x_m||) \\ &+ 2\bar{b}||\bar{K}e||(||\theta^*|| + \bar{\phi})(||r|| + ||e|| + N) \\ &- \sigma||\phi + \rho\bar{\theta}||^2 + \sigma||\theta^*||^2 \\ &\qquad \forall (t, e, \phi) \in \mathcal{R} \times \Omega. \end{aligned} \tag{7.33}$$

Completing the square involving $||\bar{K}e||$, and dropping the negative term, we finally get

$$\begin{aligned}\dot{V}(e,\phi)_{(7.21)} \leq &- \left\{\lambda_{\min}(G) - \rho^{-1}[\xi + \bar{b}(||\theta^*|| + \bar{\phi})]^2\right\} ||e||^2 \\ &+ \rho^{-1}\chi\left[||K^*|| + \xi + \bar{b}(||\theta^*|| + \bar{\phi})\right]^2 \\ &- \sigma||\phi + \rho\bar{\theta}||^2 + \sigma||\theta^*||^2 \\ &\forall(t,e,\phi) \in \mathcal{R} \times \Omega,\end{aligned} \tag{7.34}$$

where $\chi = \max\left\{\sup_t ||x_m(t)||^2,\ (\sup_t ||r_m(t)|| + N)^2\right\}$.

Equation (7.34) can be written compactly as

$$\begin{aligned}\dot{V}(e,\phi)_{(7.21)} \leq -\zeta||e||^2 - \sigma||\phi + \rho\bar{\theta}||^2 + \eta \\ \forall(t,e,\phi) \in \mathcal{R} \times \Omega,\end{aligned} \tag{7.35}$$

where the constants ζ and η are defined

$$\begin{aligned}\zeta &= \lambda_{\min}(G) - \rho^{-1}\left[\xi + \bar{b}(||\theta^*|| + \bar{\phi})\right]^2 \\ \eta &= \sigma||\theta^*||^2 + \rho^{-1}\chi\left[||K^*|| + \xi + \bar{b}(||\theta^*|| + \bar{\phi})\right]^2.\end{aligned} \tag{7.36}$$

Selecting ρ large enough so that $\zeta > 0$, we see that (7.35) implies

$$\begin{aligned}\dot{V}(e,\phi)_{(7.21)} \leq -\mu V(e,\phi) + \eta \\ \forall(t,e,\phi) \in \mathcal{R} \times \Omega,\end{aligned} \tag{7.37}$$

where the positive number μ is given by

$$\mu \leq \min\left[\lambda_{\max}^{-1}(H)\zeta,\ \lambda_{\min}(\Gamma)\sigma\right]. \tag{7.38}$$

From (7.37) we see that $\dot{V}(e,\phi)_{(7.21)} < 0$ if $(e,\phi) \in \Omega$ and $V(e,\phi) > V_f$ where

$$V_f = \mu^{-1}\eta. \tag{7.39}$$

V_f can be made as small as desired by selecting a sufficiently large ρ, sufficiently small σ and sufficiently large Γ_i. By this design, we make $V_f < V_0$, and thereby create a region $\{(e,\phi) \in \Omega : V_0 > V(e,\phi) > V_f\}$ where $\dot{V}(e,\phi)_{(7.21)} < 0$. Using

Liapunov stability theory, we can thus conclude that solution trajectories for $\hat{\mathbf{S}}_e$ that start in Ω ultimately end up in the compact set Ω_f defined by

$$\Omega_f = \{(e, \phi) \in \Omega : V(e, \phi) \leq V_f\} \tag{7.40}$$

Therefore, the solutions $(e, \phi)(t; t_0, e_0, \phi_0)$ of $\hat{\mathbf{S}}_e$ are locally ultimately bounded with respect to the bound V_f and the local region Ω. **Q.E.D.**

The choice of the constants σ, Γ_i, and ρ to satisfy the conditions of Theorem (7.22) is not immediately obvious since ξ and $\bar{\phi}$ depend on the chosen Γ. We show here a constructive technique for selecting these constants appropriately. The technique is based on first choosing σ and Γ_i under the assumption that $\rho = \infty$. Then we choose a finite value for ρ, sufficiently large that V_f is smaller than V_0.

Assume that our intention is to have

$$V_f \leq \epsilon V_0 \tag{7.41}$$

for given positive constants V_0, V_f, and $0 < \epsilon < 1$. We now choose

$$\sigma \leq \epsilon V_0 \frac{\lambda_{\min}(G)}{\lambda_{\max}(H)\|\theta^*\|^2} \tag{7.42}$$

and any $\Gamma > 0$ such that

$$\lambda_{\min}(\Gamma) \geq \frac{\|\theta^*\|^2}{\epsilon V_0} \tag{7.43}$$

and, for the moment, we let $\rho = \infty$. Substituting into (7.36)—(7.39), it follows that (7.41) is satisfied.

Given the above choices for σ and Γ, we can now calculate

$$\begin{aligned} \bar{\phi} &= V_0^{1/2} \lambda_{\max}^{1/2}(\Gamma) \\ \bar{e} &= V_0^{1/2} \lambda_{\min}^{-1/2}(H) \end{aligned} \tag{7.44}$$

and thus

$$\xi = \alpha \bar{e} + \beta. \tag{7.45}$$

Let us now choose a finite value for ρ. Let

$$\begin{aligned}\rho = \rho_0 \max\Big\{&\lambda_{\min}^{-1}(G)\big[\xi + \bar{b}(\|\theta^*\| + \bar{\phi})\big]^2, \\ &\sigma^{-1}\|\theta^*\|^{-2}\chi\big[\|K^*\| + \xi + \bar{b}(\|\theta^*\| + \bar{\phi})\big]^2\Big\}\end{aligned} \tag{7.46}$$

where ρ_0 is a finite positive number. Substituting into (7.36), we have

$$\begin{aligned}\zeta &\geq (1 - \rho_0^{-1})\lambda_{\min}(G) \\ \eta &\leq (1 + \rho_0^{-1})\sigma\|\theta^*\|^2\end{aligned} \tag{7.47}$$

and, using (7.42) and (7.43),

$$\mu = (1 - \rho_0^{-1})\frac{\sigma\|\theta^*\|^2}{\epsilon V_0} \tag{7.48}$$

Thus, from (7.39)

$$V_f \leq \frac{1 + \rho_0^{-1}}{1 - \rho_0^{-1}}\epsilon V_0. \tag{7.49}$$

Therefore, if we choose the number ρ_0 such that

$$\rho_0 > \frac{1 + \epsilon}{1 - \epsilon} \tag{7.50}$$

then $V_f < V_0$ and the constants are appropriately chosen to satisfy the requirements of Theorem (7.22).

Note that our original intention was to have $V_f \leq \epsilon V_0$. The argument above shows that this inequality can be ***very nearly*** satisfied by choosing $\rho_0 \gg 1$. To show that we can select constants to satisfy (7.41) *strictly* for any given ϵ, we select any $\rho_0 > 1$ and substitute into the formulas

$$\begin{aligned}\sigma &\leq \epsilon\frac{(1 - \rho_0^{-1})}{(1 + \rho_0^{-1})}\,\frac{V_0\lambda_{\min}(G)}{\lambda_{\max}(H)\|\theta^*\|^2} \\ \lambda_{\min}(\Gamma) &\geq \frac{(1 + \rho_0^{-1})}{(1 - \rho_0^{-1})}\,\frac{\|\theta^*\|^2}{\epsilon V_0} \\ \bar{\phi} &= V_0^{1/2}\lambda_{\max}^{1/2}(\Gamma) \\ \bar{e} &= V_0^{1/2}\lambda_{\min}^{-1/2}(H) \\ \xi &= \alpha\bar{e} + \beta \\ \rho &> \rho_0 \max\Big\{\lambda_{\min}^{-1}(G)\big[\xi + \bar{b}(\|\theta^*\| + \bar{\phi})\big]^2, \\ &\qquad \sigma^{-1}\|\theta^*\|^{-2}\chi\big[\|K^*\| + \xi + \bar{b}(\|\theta^*\| + \bar{\phi})\big]^2\Big\}\end{aligned} \tag{7.51}$$

to give the required constants, σ, Γ, and ρ. Substituting the above into (7.36)—(7.39), we see that equation (7.41) is now strictly satisfied ($V_f < \epsilon V_0$). Thus we have proven by construction that a local stability region and residual set exist, and that the size of the residual set can be made arbitrarily small by appropriate choice of design constants.

(7.52) **Example.** As a simple example, consider the $N = 2$ case. Assume that the constants describing the plant and interconnection strengths are $k_{0i}^* = 1$, $\alpha = 1/100$, $\beta = 1/100$, $\bar{b} = 1/100$. Let

$$\begin{aligned} A_{mi} &= \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}, \quad b_{mi} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ G_i &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad H_i = \begin{pmatrix} 1.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \end{aligned} \qquad i = 1, 2 \tag{7.53}$$

as in Chapter 3. Assume, for our example here, that this implies $||\theta^*|| = 1$, and also assume that the reference model trajectory obeys $\sup_t ||x_m(t)|| = 1$ and $\sup_t ||r_m(t)|| = 1$. Thus $\chi = 9$. Choose $V_0 = 100$, $V_f = 1$ (thus $\epsilon = 0.01$), and $\rho_0 = 2$. Using (7.51) we have

$$\begin{aligned} \sigma &= 0.195 \\ \Gamma &= 3.00I \\ \bar{\phi} &= 17.3 \\ \bar{e} &= 18.5 \\ \xi &= 0.195 \\ \rho &= 175 \end{aligned} \tag{7.54}$$

which completes our example.

(7.55) **Remark**. As in the state feedback case covered in Chapter 3, we find that there is a tradeof between small tracking errors and large gains. This time, however, it is necessary for the designer to specify ρ, since it is used in the adaptation law (7.20).

(7.56)**Remark**. Many commercial robot arms use servo motors that drive the joints through a series of gears. If the gear ratio is high, then the inertia matrix $J(q, l)$ will be dominated by large elements along the diagonal. The net effect is that the subsystem interconnection strengths, represented by the constants α, β, $\bar{b}$ will be very small: on the order of the inverse of the gear ratio squared. For example, the PUMA robot arm (described in the next chapter) has gear ratios of approximately 100. In such cases, the decentralized adaptive control design will require only a very small value for ρ. In direct drive arms, such as Carnege-Mellon's DDII, there are no gears (the "gear ratio" equals one) so the interconnections are quite strong and a relatively high value for ρ is required.

(7.57) **Remark**. The method (7.51) of choosing constants for Theorem (7.22) can not be used as a design technique when we presume that *a-priori* information about the plant parameters θ^* and interconnection strengths α, β, and $\bar{b}$ is not available. In practice, however, upper bounds on these values may be known, and rational design choices must be made. An understanding of the roles of the design constants can help the designer to adjust them. The designer should keep in mind the fact that Γ_i acts as an adaptive "attack" constant, determing the rate of gain adaptation per given level of tracking error. Also, σ is a "decay" constant, which provides a fading memory for the adaptive mechanism. Finally, ρ is the high gain parameter, with $\rho\bar{\theta}_i$ being the nominal gain setting around which adaptation occurs.

8
Experiments with the PUMA arm

In this chapter we present the results of motion control experiments with the PUMA robot arm at the U. C. Davis Robotics Research Laboratory. We controled the robot arm using the high gain decentralized adaptive controller presented in this thesis, and also, in comparison experiments, we controlled the arm using a non-adaptive decentralized control scheme. We did not experiment with any centralized control laws because of the limited capability of the on-line control computer.

The results from three PUMA arm motion experiments are presented and discussed below. We selected arm trajectories which tend to induce a high degree of dynamic coupling between the joints thus testing the ability of the decentralized adaptive controller to reject interconnection disturbances. The first experiment was designed to demonstrate the torque intercoupling due to centrifugal forces $c_i(q, \dot{q}, l)$. The last two experiments demonstrate inertial intercoupling, that is, due to the off diagonal elements of $J(q, l)$.

Figure 8.1 is a drawing of the PUMA arm, indicating the rotation senses of each of the six joints. We are concerned here with the first three joints, waist, elbow, and shoulder, since they exhibit the highest degree of intercoupling due to large link inertias and non-orthogonal rotation axes. Figure 8.2 shows the arm in the zero configuration (straight up). The arm is driven by servo motors mounted at each joint, and joint position is read from an optical encoder mounted on each motor shaft. Joint velocities must be computed within the control processor given periodic position information.

We have modified the factory provided PUMA system by replacing the control processor with our own INTEL 286 processor and UNIX operating system, allowing us to easily program experimental control algorithms [57]. The experiments described below were run at a sample rate of 100 hertz.

The optically encoded position signal has a resolution of one count. There are approximately 60,000 index counts over the full range of motion for each joint, thus one count corresponds to approximately $2\pi/60,000 = 0.1$ milliradian of joint motion. We will use these counts as the units of joint position in the disscussion that follows. The motors are DC servos with 300 ounce-inch maximum torque. The control signal from the computer is from a digital to analog converter which translates an integer ranging from -32,000 to +32,000 to a motor drive voltage that ranges from -40 to +40 volts, giving a full range of available torque. We will use the control integer from the computer as the unit for control signals. Keep in mind that saturation occurs at $\pm 32,000$.

The experiment trajectories are shown in Figure 8.3 a, b, and c. Each trajectory was run in both an adaptive and a non-adaptive mode, and in all cases, both control laws caused the arm to follow the given trajectory within a few hundred counts, which is indistinguishable on the scale of Figure 8.3.

For the adaptive experiments, tracking error is defined by

$$e_i(t) = q_i(t) - q_{mi}(t) \qquad i \in \{1,2,3\}. \tag{8.1}$$

where q_i is the i'th robot joint angle and q_{mi} is the i'th joint angle in the reference model. The reference model can be written in the second order form as

$$\ddot{q}_{mi} + a^d_{mi}\dot{q}_{mi} + a^p_{mi}q_{mi} = r_{mi}(t), \qquad i \in \{1,2,3\} \tag{8.2}$$

which corresponds to the first order form (7.15) with $x_{mi} = (q_{mi}, \dot{q}_{mi})^T$ and

$$A_{mi} = \begin{pmatrix} 0 & 1 \\ -a^p_{mi} & -a^d_{mi} \end{pmatrix} \quad b_{mi} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad i \in \{1,2,3\}. \tag{8.3}$$

The adaptive control law is given by

$$\tau_i(t) = -k^p_i(t)e_i(t) - k^d_i(t)\dot{e}_i(t) - k^i_i(t) \qquad i \in \{1,2,3\} \tag{8.4}$$

where $\tau_i(t)$ is the control torque for joint i. The rules for adapting $k_i^p(t)$, $k_i^d(t)$, and $k_i^i(t)$ are

$$\begin{aligned}
\dot{k}_i^p &= \gamma_i^p \bar{e}_i(t) e_i - \sigma_i^p \gamma_i^p [k_i^p(t) - k_i^p(0)] \\
\dot{k}_i^d &= \gamma_i^d \bar{e}_i(t) \dot{e}_i - \sigma_i^d \gamma_i^d [k_i^d(t) - k_i^d(0)] \qquad i \in \{1,2,3\} \\
\dot{k}_i^i &= \gamma_i^i \bar{e}_i(t) - \sigma_i^i \gamma_i^i [k_i^i(t) - k_i^i(0)]
\end{aligned} \tag{8.5}$$

where

$$\bar{e}_i(t) = \bar{k}_i^p e_i(t) + \bar{k}_i^d \dot{e}_i(t) \qquad i \in \{1,2,3\}, \tag{8.6}$$

$k_i^p(0)$, $k_i^d(0)$, and $k_i^i(0)$ are initial settings for the adapted gains, γ_i^p, γ_i^p, γ_i^i, σ_i^p, σ_i^d, and σ_i^i are positive design constants, and $\bar{k}_i^p$ and $\bar{k}_i^d$ are constants chosen by the designer so as to satisfy (3.3) and (3.4), where $\bar{k}_i = (\bar{k}_i^p, \bar{k}_i^d)^T$. We chose

$$a_{mi}^p = 225, \quad a_{mi}^d = 30, \qquad i \in \{1,2,3\} \tag{8.7}$$

so that each joints reference model has a double pole at $s = -15\ \text{sec}^{-1}$. With this choice, equations (3.3) and (3.4) are satisfied with the positive definite matrices

$$G_i = \begin{pmatrix} 6750 & 0 \\ 0 & 30 \end{pmatrix}, \quad H_i = \begin{pmatrix} 675 & 15 \\ 15 & 1 \end{pmatrix}. \qquad i \in \{1,2,3\} \tag{8.8}$$

and

$$\bar{k}_i^p = 15, \quad \bar{k}_i^d = 1 \qquad i \in \{1,2,3\}. \tag{8.9}$$

The adaptation constants were selected, using the heuristic rules outlined in remark (7.52), as

$$\begin{aligned}
\gamma_i^p &= 0.8, \quad \gamma_i^d = 0.02, \quad \gamma_i^i = 1.0, \\
\sigma_i^p &= 5.6, \quad \sigma_i^d = 7.7, \quad \sigma_i^i = 0.
\end{aligned} \qquad i \in \{1,2,3\} \tag{8.10}$$

In the nonadaptive experiments, a proportional integral derivative (PID) controller was used. The tracking error in this case is defined as

$$e_i(t) = q_i(t) - q_{d_i}(t) \qquad i \in \{1,2,3\} \tag{8.11}$$

where $q_{di}(t)$ is the desired trajectory. The PID controller's desired trajectory is simply a scaled version of the adaptive controller's reference model input, $q_{di}(t) = r_{mi}/a^p_{mi}$. This is so that the desired robot arm motion is the same for both types of controllers. The PID control law for each joint is given by

$$\begin{aligned} \tau_i &= -k_i^p e_i - k_i^d \dot{e}_i - k_i^i f_i \\ \dot{f}_i &= e_i \end{aligned} \qquad i \in \{1,2,3\} \tag{8.12}$$

where k_i^p, k_i^i, and k_i^d are the fixed proportional, integral and derivative gains, respectively. The gain settings were determined experimentally by a graduate student who is a veteran at tuning PID laws from past work experience:

$$\begin{aligned} k_1^p &= 130, \quad k_2^p = 180, \quad k_3^p = 110 \\ k_1^d &= 270, \quad k_2^d = 270, \quad k_3^d = 120 \\ k_1^i &= 0.7, \quad k_2^i = 0.7, \quad k_3^i = 0.7 \end{aligned} \tag{8.13}$$

The values shown above were also used for the initial gains $k_i^p(0)$ and $k_i^d(0)$ in the adaptive experiments. Before each experiment, the robot was held motionless for a few seconds to allow the integrator states (k_i^i in the adaptive law, and f_i in the non adaptive law) to settle. This was to ensure consistency in comparison runs.

We now describe each of the experiments in turn.

Experiment 1 is a high speed twisting motion of the waist joint with the shoulder and elbow joints maintained in the L position. The trajectory is plotted in Figure 8.3(a). The starting arm position is shown in Figure 8.4. The waist motion induces a centrifugal force on the forearm link, causing a disturbance torque at the elbow joint. (Note that during this experiment the elbow joint was actually moving very slowly. This was done in order that restoring torques due to static friction in the elbow gear train do not dominate torques due to centrifugal force on the forearm. Experiments with the elbow held steady revealed exactly zero tracking error at the elbow joint for both adaptive and non-adaptive control laws.) Refering to plots of tracking error in Figure 8.5(a), we see that adaptation causes the elbow joint to track more accurately. Figure 8.5(b) shows the position feedback gain adjustments. Note that, in addition to centrifugal forces, the abrupt halt in waist motion at $t = 0.8$ seconds (see Figure 8.3(a)) caused a brief disturbance pulse (Figure 8.5(a), at $t = 0.8$ seconds). The disturbance is more evident in the tracking error associated with non adaptive control than with adaptive control.

Experiments 2 and 3 are designed to demonstrate that control torques at the elbow joint are coupled to the shoulder. Even though shoulder motion is identical in both cases, shoulder tracking performance can be quite different depending on the elbow motion. Adaptation helps to decouple the motions.

Experiment 2 is a lifting motion with both the shoulder and elbow joint starting from the straight out position (Figure 8.3(b) shows the trajectory and Figure 8.6 shows the starting position). The combination of gravity forces and the torque necessary at the elbow joint to lift the forearm causes a large disturbance torque, in the positive rotation direction, at the shoulder joint. Tracking results for both adaptive and non-adaptive controllers are shown in Figure 8.7(a). Note that the adaptive controller is much better at canceling the disturbance torques, and does so by increasing the feedback gains (Figure 8.7(b)).

Experiment 3 is identical to experiment 2, except that the forearm (elbow joint) is dropped instead of lifted (Figure 8.3(c) shows the trajectory and Figure 8.8 shows the starting position). In this case, the torque applied at the elbow joint is coupled to the shoulder joint in the negative rotation direction. This tends to cancel the gravity disturbance which is felt in the positive rotation direction. The result is that the total disturbance torque at the shoulder is small. From Figure 8.9(a) we see that both the non-adaptive and adaptive controllers have about the same tracking error performance and that adaptive gains (Figure 8.9(b)) adjust to much lower values than before.

Through these experiments, we have demonstrated the feasibility of decentralized adaptive control of a robot arm. The experimental results show that the adaptation mechanism tends to decouple the joint dynamics through the use of high feedback gains, as predicted by the analysis of the earlier chapters. For example, in experiment 2, when the motion of the elbow joint strongly interacted with the motion of the shoulder joint, the shoulder joint gains increased to help keep the shoulder tracking error small.

We have also shown that the adaptation mechanism refrains from using high gains when they are not necessary. This behavior was demonstrated in experiment 3, where the trajectory happened to be one where disturbance torques canceled and gains remained at modest values.

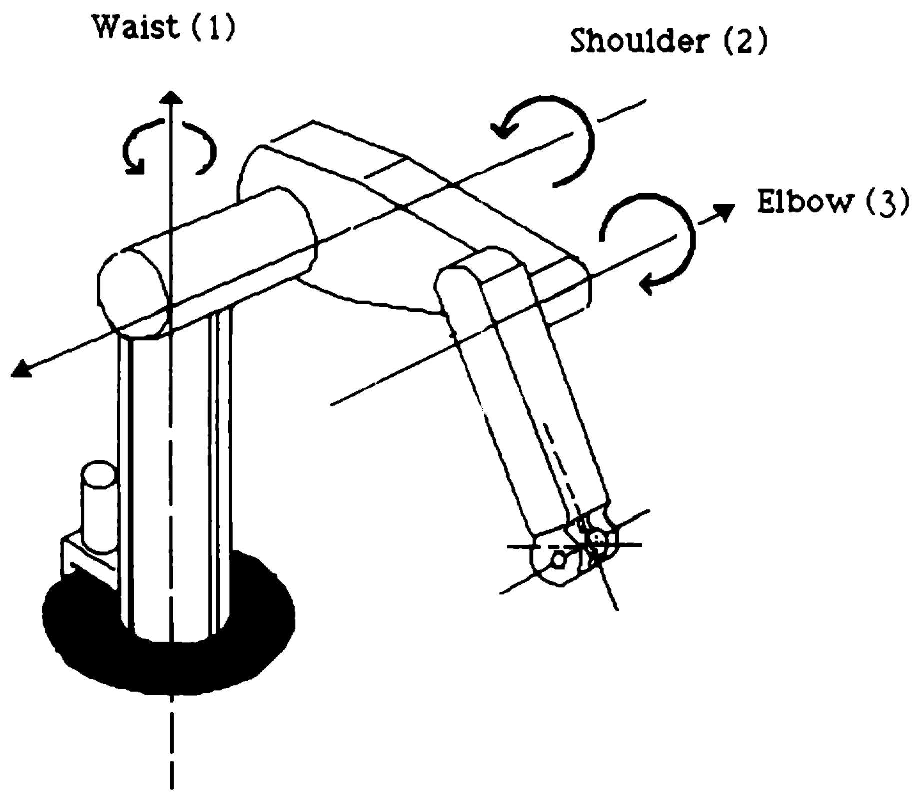

Figure 8.1. PUMA robot arm.

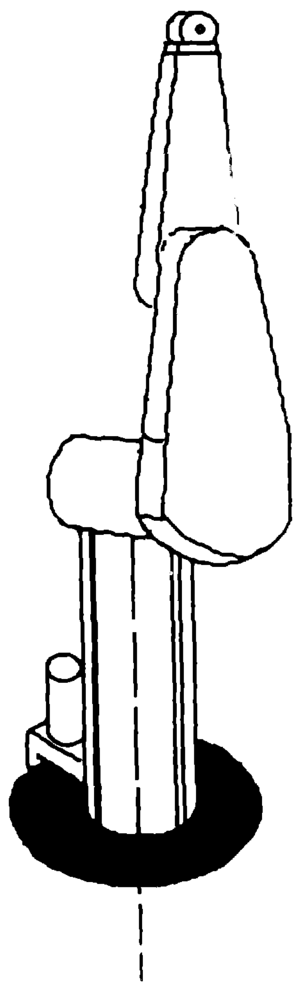

Figure 8.2. PUMA arm in zero configuration.

a

b

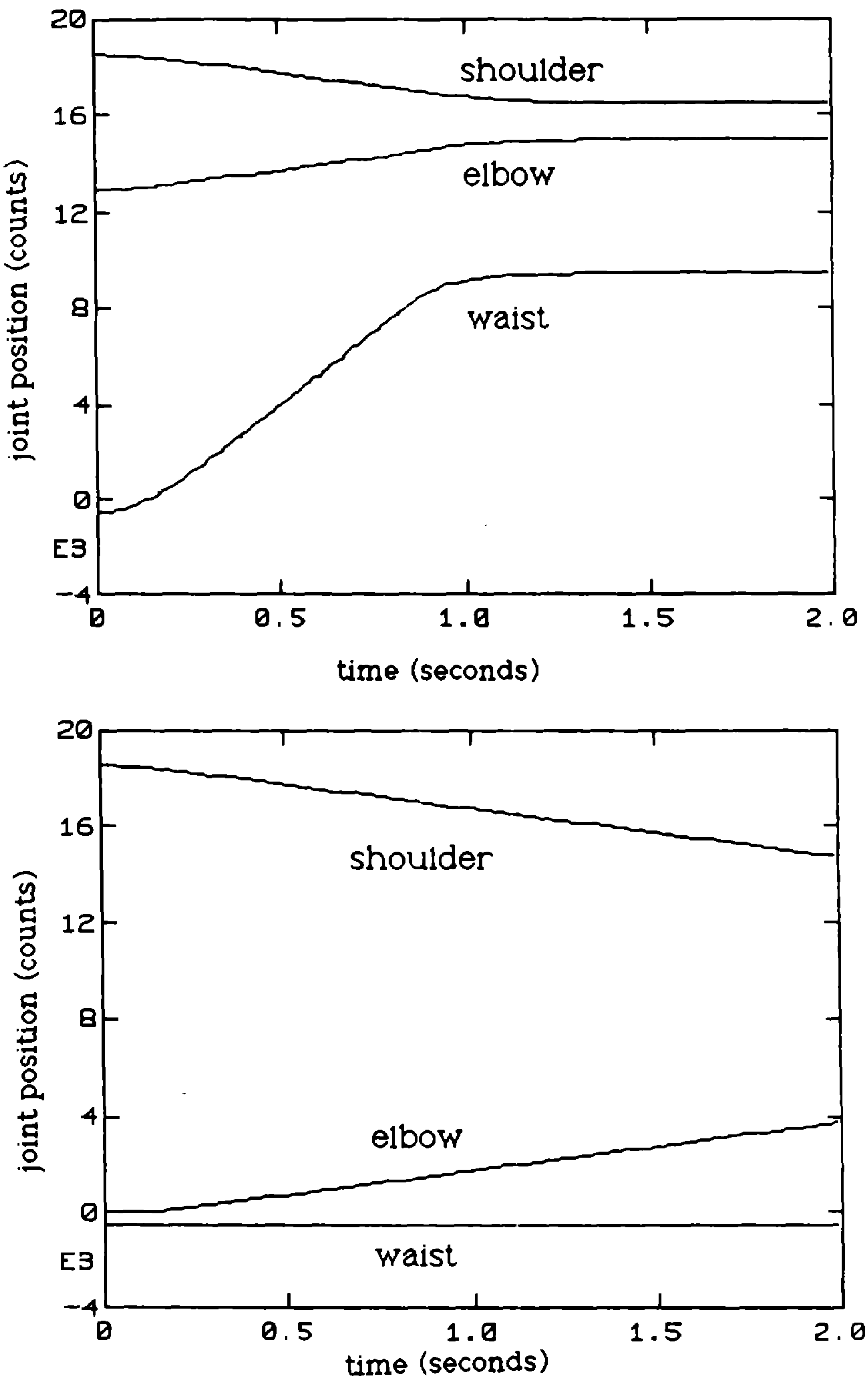

Figure 8.3. Experimental trajectories, $q_{mi}(t)$: a) Experiment 1, b) Experiment 2.

C

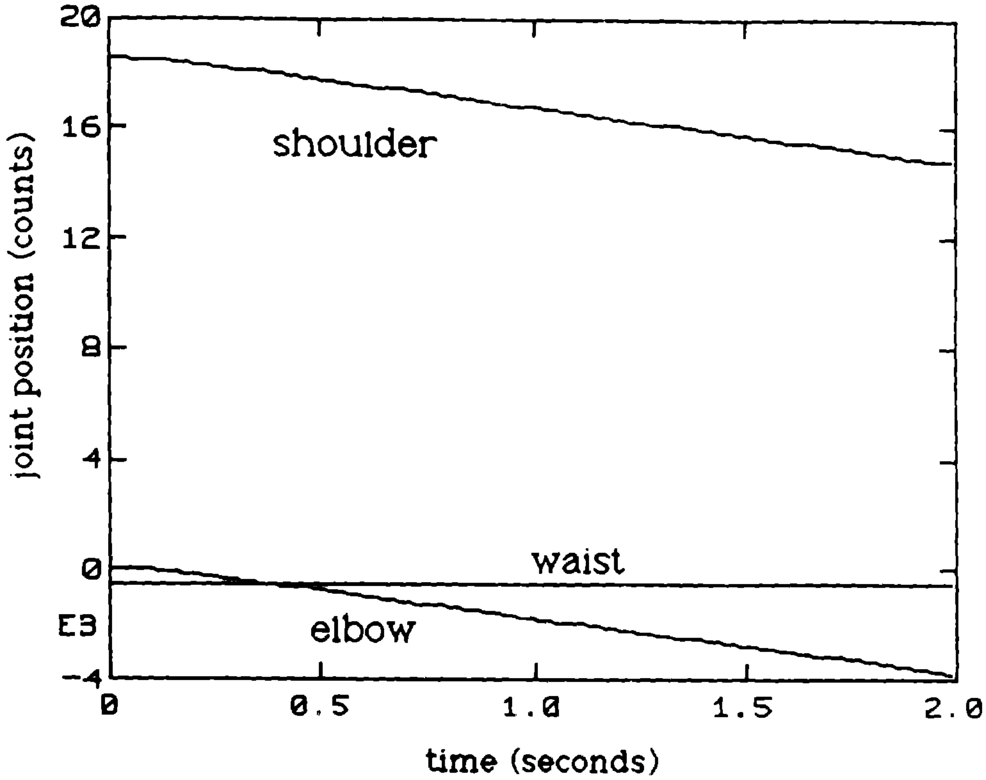

Figure 8.3(continued). Experimental trajectories, $q_{mi}(t)$: c) Experiment 3.

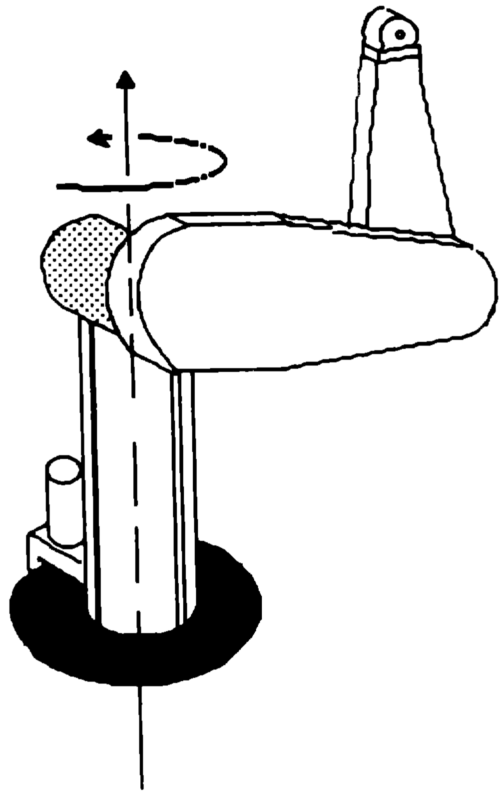

Figure 8.4. Experiment 1: waist twist.

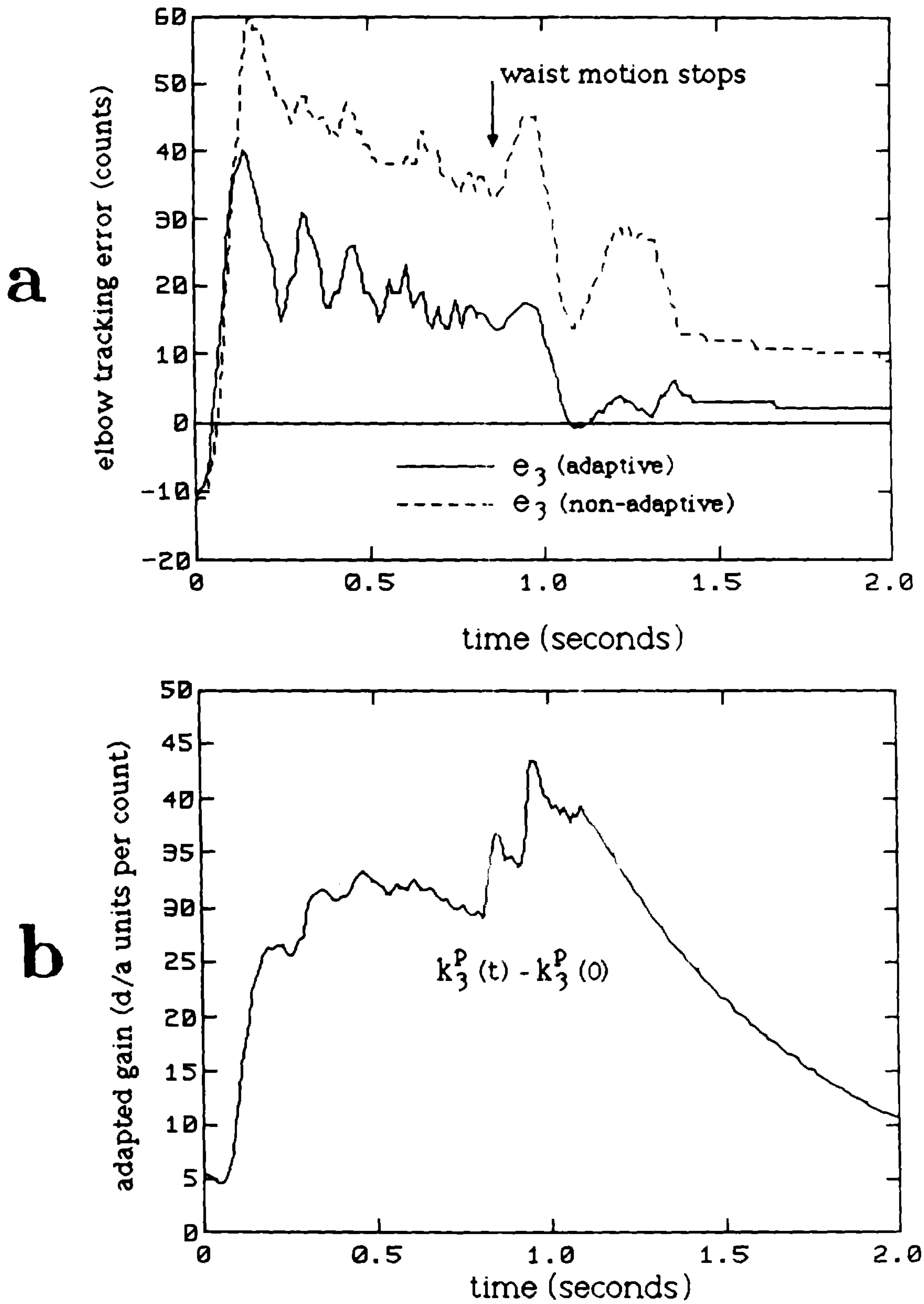

Figure 8.5. Experiment 1: a) tracking errors. b) position gains.

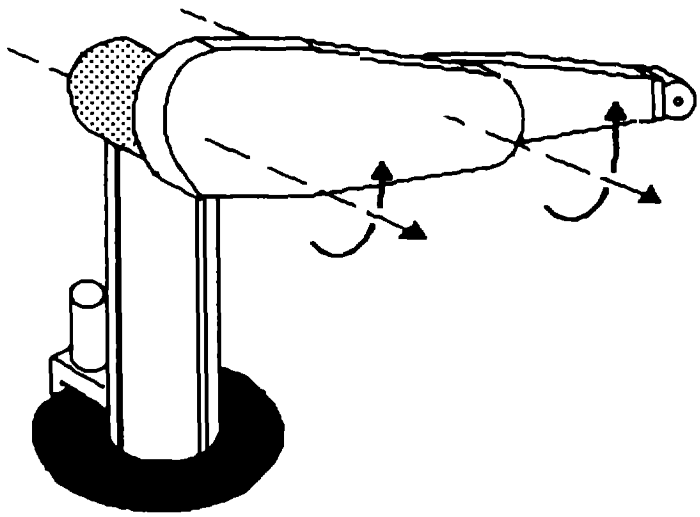

Figure 8.6. Experiment 2: lifting motion.

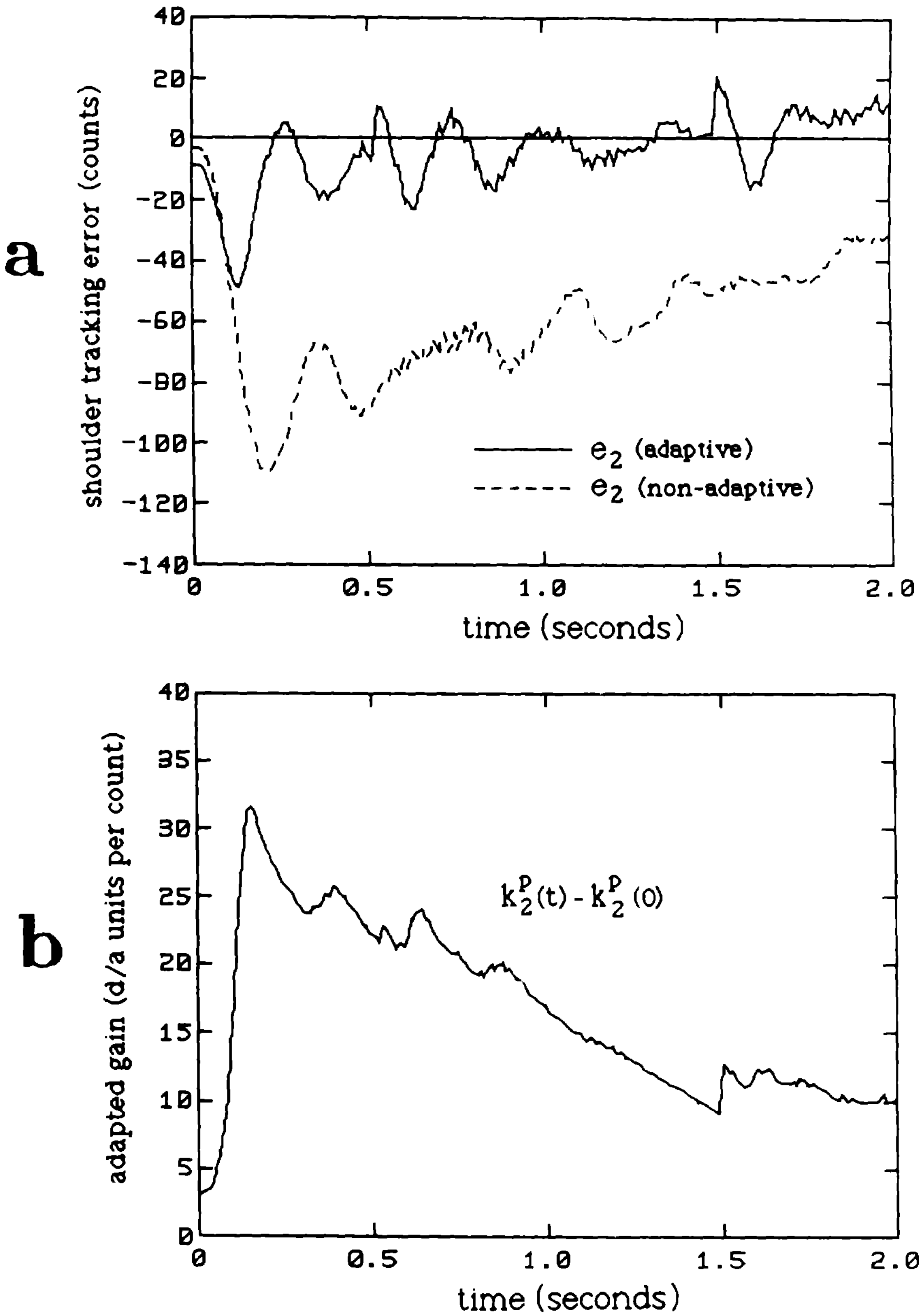

Figure 8.7. Experiment 2: a) tracking errors b) position gains.

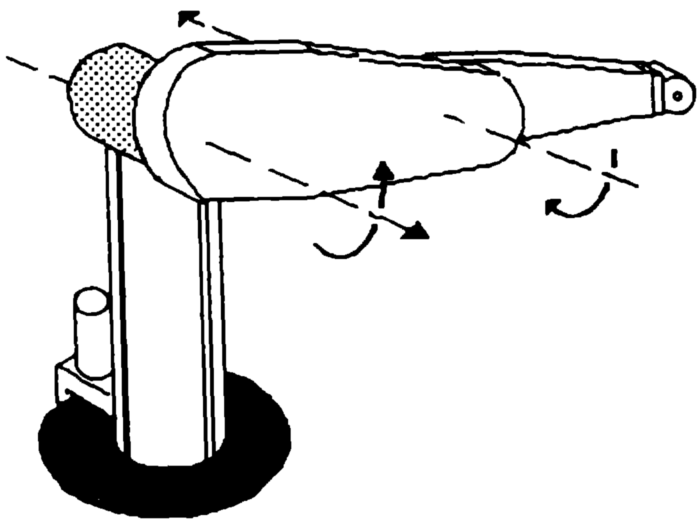

Figure 8.8. Experiment 3: dropping motion.

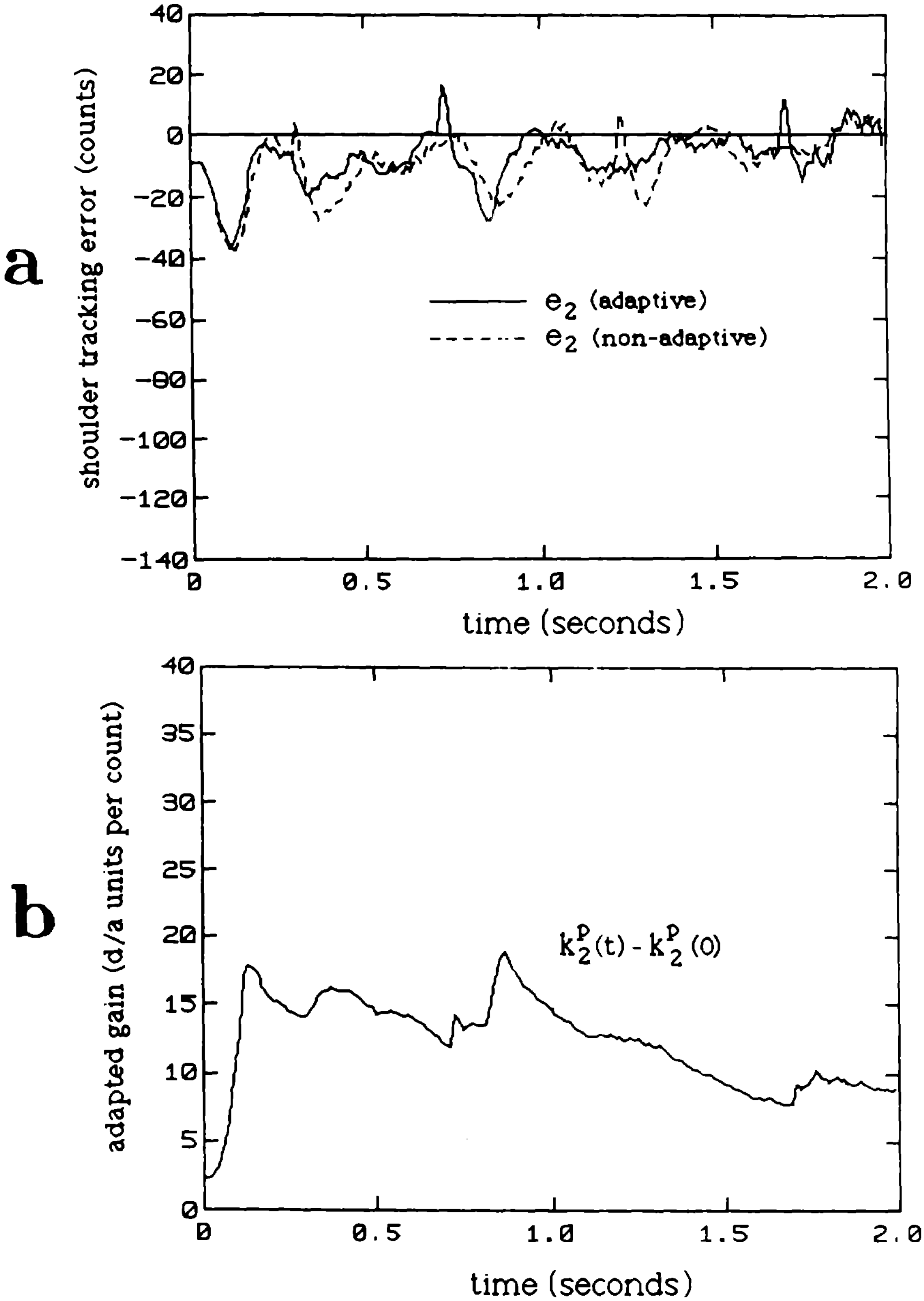

Figure 8.9. Experiment 3: a) tracking errors b) position gains.

9 Conclusion

In this thesis we have developed decentralized adaptive controllers for several classes of nonlinear large scale interconnected systems. The theory is been based on the high gain stabilization techniques employed in [3-7] for decentrally stabilizable systems. The contribution of this thesis is the on-line *adaptive* implementation of these techniques using a minimum of *a-priori* knowledge about the plant.

We first focused on an important subset of the decentrally stabilizable nonlinear systems, those where the interconnection disturbances enter the subsystems through the range space of the controls. We derived the full state feedback decentralized adaptive *regulator* and discovered that it is the same law as that proposed by Ioannou [22]. The decentralized model-following *tracker*, however, requires a different set of regressors (3.22) in order to assure stability in the presence of strong interconnections. Simulations of a strongly coupled dual pendulum system demonstrated the superior tracking performance of the newly proposed algorithm.

An important result of the analysis of this class of decentralized adaptive control laws is that stability is assured *regardless of interconnection strengths*. Feedback gains automatically adjust to whatever level is necessary in order to cancel the disturbance sufficiently.

Next we presented an adaptive controller for the special case where subsystem dynamics are known, but interconnection strengths are not. In this case, only one adapted parameter per subsystem is necessary. As to be expected from using increased knowledge of the system, simulations showed better tracking response with this algorithm than with the other ones that assumed no knowledge of the plant.

An extension of the adaptive tracking algorithms to the output feedback case was then presented. We derived a stable decentralized control law for the class of output-feedback decentrally stabilizable systems studied in [7]. The control law employs pre- and post- compensators to make up for the lack of full state information. In closed loop, the presence of the prefilter upsets the range space assumption exploited in the state feedback case. This range condition is recovered however by reflecting interconnection disturbances to the input of the closed loop system through the stable *inverse* of the prefilter. Overall stability can then be proved using a heirarchically coupled subsystems approach. Simulations of the output feedback controller show similar steady state tracking error as in the state feedback case but, during the initial transient period, tracking errors are worse than before, probably due to the fact that the pre- and post- compensators require some time to settle.

In Chapter 6 we introduced adaptive controllers to the wider class of decentrally stabilizable large scale systems reported in [6]. Since in this case, we relaxed the earlier range space condition, it was necessary to make more *a-priori* knowledge assumptions in order to derive a stable adaptive control law. In particular, it was found necessary that the designer provide sufficiently stable reference models to assure overall stability. This choice must take into account upper bounds on interconnection strengths.

Finally, we derived a decentralized adaptive state feedback control law for robot motion control. It was shown that the natural subsystems of a robot manipulator, which are the individual joints with their respective servo motors, do not form an *input-decentralized* system, that is, control torques applied to one joint cross couple as disturbance torques at other joints. We then formulated a locally stable decentralized adaptive control law for the non input-decentralized systems and gave an example for choosing design constants to assure a finite stability region.

The robot arm adaptive controller of Chapter 7 has been implemented on a PUMA arm at the U. C. Davis Robotics Research Laboratory. Experiments have shown that the varying interconnection strengths due to changing arm configuration and joint velocities can signifigantly affect the independent joint dynamics. For this reason, local PID loops with fixed gains will result in varying tracking performance. Under adaptive control, the arm exhibited a more decoupled behavior, since adapted gains increased to cancel the strong interconnection disturbances, and decreased when the distrubances were small

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A
Large Scale Systems

In this appendix, we provide a tutorial introduction to the theory of large scale systems. The material presented here forms the basis of the theory developed in the main part of the thesis. The reader is also encouraged to consult the relevant texts in the area [25,58] for more details.

The design of a decentralized controller has two principle stages. First the large scale system must be decomposed into component subsystems. Second the local controllers for the subsystems must be designed so as to stabilize the overall system. The first step, that of decomposition, can be accomplished along the natual divisions occuring within the system, that is, between the obvious subsystems which arise in the model. For example, in the case of a robot arm, each joint, with its servo motor, is a natural subsystem. Alternatively, mathematical subsystems can be created by grouping system equations so as to give the most convenient interconnection structure. Such an approach been studied extensively [59,60]. An important goal of the decomposition process is to make the subsequent job of control design as easy as possible. The interconnection structure, as well as the strength of the interconnections, will determine whether the resulting decompostion is decentrally controllable.

This thesis is concerned mostly with the second step of the process, that of local controller design. We seek to find control laws which, although applied locally, using only locally available information, will nevertheless stabilize the overall interconnected large scale system.

We consider the large scale system $\mathbf{S}$ as consisting of an interconnection of N smaller subsystems $\mathbf{S}_i$, where $i = 1, 2, \ldots, N$. Denoting the index set $\boldsymbol{N} =$

$\{1, 2, \ldots, N\}$, the interconneced system is described by

$$\mathbf{S}_i : \quad \begin{aligned} \dot{x}_i &= A_i x_i + P_i f_i(t, x) + B_i u_i \\ y_i &= C_i x_i , \end{aligned} \qquad i \in \boldsymbol{N} \tag{A.2}$$

where $x_i \in \mathcal{R}^{n_i}$ are the subsystem state vectors, $y_i \in \mathcal{R}^{l_i}$ are the measurable subsystem outputs, $u_i \in \mathcal{R}^{m_i}$ are the available subsystem control inputs, and A_i, B_i and P_i are constant matrices of dimensions $n_i \times n_i$, $n_i \times m_i$, and $n_i \times p_i$ respectively. We define $x = (x_1^T, x_2^T, \ldots x_N^T)^T \in \mathcal{R}^n$ as the overall system state vector, and assume that the functions $f_i : \mathcal{R} \times \mathcal{R}^n \rightarrow \mathcal{R}^{p_i}$ are sufficiently smooth so that the solution $x(t : t_0, x_0)$ of $\mathbf{S}$ is unique for all initial conditions $(t_0, x_0) \in \mathcal{R} \times \mathcal{R}^n$ and all piece-wise continuous inputs $u(\cdot)$. It is assumed that the functions $f_i(t, x)$ obey the inequalities

$$\|f_i(t, x)\| \leq \sum_{j=1}^{N} \eta_{ij} \|x_j\| \qquad \forall (t, x) \in \mathcal{R} \times \mathcal{R}^n, \, i \in \boldsymbol{N} \tag{A.3}$$

for some positive constants η_{ij}, $i, j \in \boldsymbol{N}$.

It is desired to stabilize the overall system $\mathbf{S}$ with local feedback control. If the full state of the system is available, the control law is of the form

$$u_i = -K_i x_i \qquad i \in \boldsymbol{N} \tag{A.4}$$

where K_i is a constant $m_i \times n_i$ matrix This results in the closed-loop system

$$\hat{\mathbf{S}}_i : \quad \begin{aligned} \dot{x}_i &= \hat{A}_i x_i + P_i f_i(x) \\ y_i &= C_i x_i \end{aligned} \qquad i \in \boldsymbol{N}. \tag{A.5}$$

where $\hat{A}_i = (A_i - B_i K_i)$. If only the outputs, y_i, are available for feedback, we use dynamic compensators of the form

$$\mathbf{C}_i : \quad \begin{aligned} \dot{z}_i &= F_i z_i + R_i y_i \\ u_i &= -S_i z_i - D_i y_i \end{aligned} \qquad i \in \boldsymbol{N} \tag{A.6}$$

where $z_i \in \mathcal{R}^{r_i}$, and F_i, R_i, S_i, and D_i are matrices of appropriate dimension. This results in the closed-loop dynamic system

$$\hat{\mathbf{S}}_i : \quad \begin{array}{l} \dot{\hat{x}}_i = \hat{A}_i \hat{x}_i + \hat{P}_i f_i(t, x) \\ y_i = \hat{C}_i \hat{x}_i \end{array} \qquad i \in N \tag{A.7}$$

where $\hat{x}_i = (x_i^T, z_i^T)^T$, and

$$\begin{aligned} \hat{A}_i &= \begin{pmatrix} A_i - B_i D_i C_i & -R_i H_i \\ S_i C_i & F_i \end{pmatrix} \\ \hat{P}_i &= \begin{pmatrix} P_i \\ 0 \end{pmatrix}, \\ \hat{C}_i &= \begin{pmatrix} C_i & 0 \end{pmatrix} \end{aligned} \tag{A.8}$$

The control design process involves the proper choice of state feedback matrices, K_i, or compensator matrices F_i, R_i, S_i, and D_i.

For the purpose of discussion, we focus attention on the case where state feedback is used. The following arguments apply equally as well to the feedback compensator case, however, simply by substituting $\hat{x}_i$ for x_i, $\hat{P}_i$ for P_i, etc. in the equations below. To determine whether or not the large scale system, $\hat{\mathbf{S}}$ is stable, we introduce the vector Liapunov function $v(x) = (v_1(x_1), v_2(x_2), \ldots, v_N(x_N))^T$, where each scalar function $v_i : \mathcal{R}^{n_i} \to \mathcal{R}$ is a positive definite function of the subsystem state, $\hat{x}_i$:

$$v_i(x_i) = x_i^T H_i x_i \quad i \in N \tag{A.9}$$

where H_i are positive definite $n_i \times n_i$ solution matrices to the Liapunov equations

$$\hat{A}_i^T H_i + H_i \hat{A}_i = -G_i \quad i \in N \tag{A.10}$$

for any given positive definite matrices G_i.

Next, we form a candidate scalar Liapunov function

$$V(x) = d^T v(x) \tag{A.11}$$

where $d = (d_1, d_2, \ldots, d_N)^T$ is a vector of positive constants. The system $\hat{\mathbf{S}}$ is then globally asymptotically stable if a vector d can be found such that $V(x)$ is a Liapunov function for the system $\hat{\mathbf{S}}$, that is, $d^T \dot{v}(x) < 0$ for all $(t, x) \in \mathcal{R} \times \mathcal{R}^n$.

The advantage of the vector Liapunov approach is that each $v_i(x_i)$ represents an aggregation of that subsystem's behavior. The linear combination of these aggregates, $V(x) = d^T v(x)$, then forms the aggregation of overall system's behavior. The d_i are free parameters to be chosen in any way possible to make $V(x)$ a Liapunov function.

Proceeding now to the selection of d, we take the total derivative of $V(x)$ along solutions of $\hat{\mathbf{S}}$ (A.5) to get

$$\dot{V}(x)_{(A.5)} = \sum_{i=1}^{N} d_i \big[-x_i^T G_i x_i + 2x_i^T H_i P_i f_i(x) \big] \quad \forall x \in \mathcal{R}^n. \tag{A.12}$$

Substituting inequality (A.3) we get

$$\dot{V}(x)_{(A.5)} \leq \sum_{i=1}^{N} d_i \big[-\lambda_{\min}(G_i) \|x_i\|^2 + \sum_{j=1}^{N} 2\|H_i P_i\| \eta_{ij} \|x_i\| \|x_j\| \big] \quad \forall x \in \mathcal{R}^n, \tag{A.13}$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of the indicated matrix, and $\|\cdot\|$ denotes the Euclidean norm of a vector, or the spectral norm if the argument is a matrix. Inequality (A.13) can be rewritten as

$$\dot{V}(x)_{(A.5)} \leq 2 \sum_{i=1}^{N} d_i \lambda_{\max}(H_i) \big[-\pi_i \|x_i\|^2 + \sum_{j=1}^{N} \xi_{ij} \|x_i\| \|x_j\| \big] \quad \forall x \in \mathcal{R}^n \tag{A.14}$$

where

$$\pi_i = \frac{1}{2} \frac{\lambda_{\min}(G_i)}{\lambda_{\max}(H_i)} \quad i \in N \tag{A.15}$$

and

$$\xi_{ij} = \|H_i\| \eta_{ij} \quad i, j \in N. \tag{A.16}$$

We now define the $N \times N$ matrix, $W = [w_{ij}]$ by

$$w_{ij} = \begin{cases} \pi_i - \xi_{ii} & i = j \\ -\xi_{ij} & i \neq j \end{cases} \quad i, j \in N \tag{A.17}$$

and define $\mathbf{x} = (\|x_1\|, \|x_2\|, \ldots, \|x_N\|)^T$. Inequality (A.14) now becomes

$$\dot{V}(x)_{(A.5)} \leq -\mathbf{x}^T (DW + W^T D)\mathbf{x} \quad \forall x \in \mathcal{R}^n \qquad (A.18)$$

where $D = \mathrm{diag}\{d_1 \lambda_{\max}(H_1), d_2 \lambda_{\max}(H_2), \ldots, d_N \lambda_{\max}(H_N)\}$.

In order to make the right hand side of (A.18) negative definite, we need to look for conditions under which a positive diagonal matrix D can be found to make the matrix $H = (DW + W^T D)$ positive definite. Such a D can always be constructed if W is a $\mathcal{M}$ matrix, as defined in [25]. We summarize below the properties of $\mathcal{M}$ matrices, then prove the assertion that H is positive definite if W is an $\mathcal{M}$ matrix.

Let W be a square matrix with all off diagonal elements being non-positive. W is defined to be an $\mathcal{M}$ matrix if it satisfies the property:

Property 1. The Leading principal minor determinants of W are all positive, that is

$$\begin{vmatrix} w_{11} & w_{12} & \ldots & w_{1k} \\ w_{21} & w_{22} & \ldots & w_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ w_{k1} & w_{k2} & \ldots & w_{kk} \end{vmatrix} > 0 \quad \forall k \in N \qquad (A.19)$$

Property 1 is equivalent to each of the following properties [25].

Property 2. There is a positive vector $g > 0$ such that $g' = Wg > 0$ (by $g > 0$ we mean that each element of the vector is positive)

Property 3. There is a positive vector $h > 0$ such that $h' = W^T h > 0$.

Property 4. The real part of each eigenvector of W is positive.

We can now show that if W is an $\mathcal{M}$ matrix, $H = DW + W^T D$ is positive definite. The following proof is due to Araki and Kondo [61]. According to properties 2 and 3, there are two positive vectors $g > 0$ and $h > 0$ so that $g' = Wg > 0$ and $h' = W^T h > 0$. We set $d_i = \lambda_{\max}^{-1}(H_i)h_i/g_i$, where $h_i > 0$ and $g_i > 0$ are the i'th components of h and g respectively. Then

$$Hg = Dg' + h'. \tag{A.20}$$

By construction, H is symmetric and has non-negative off diagonal elements. The right hand side of (A.20) is positive, therefore Hg is positive. Property 2 is thus satisfied and H is therefore an $\mathcal{M}$ matrix. Since H is symmetric, it has all real eigenvalues. Because of property 4 the eigenvalues of H are all positive, therefore, H is positive definite.

With H being positive definite, the right hand side of inequality (A.18) is negative, and thus $V(x)$ is a Liapunov function for $\hat{\mathbf{S}}$. Thus a sufficient condition for stability of the decentrally controlled large scale system $\hat{\mathbf{S}}$ is that W is an $\mathcal{M}$ matrix.

We may not always be able to find local feedback gains K_i such that W is an $\mathcal{M}$ matrix. This is usually a hit or miss design process. However, there are some situations where stabilizing designs can be directly constructed. One of these is where interconnection disturbances enter the range space of the control variables, that is, in (A.2) the matrix P_i can be factored as

$$P_i = B_i \tilde{P}_i \quad i \in \boldsymbol{N} \tag{A.21}$$

In that case, we use the control

$$u_i = -K_i x_i - \rho B_i^T H_i x_i \quad i \in \boldsymbol{N} \tag{A.22}$$

where ρ is a positive design constant to be assigned below. We now let the candidate Liapunov function be defined

$$V_i(x) = \sum_{i=1}^{N} v_i(x_i) \tag{A.23}$$

and take its total time derivative with respect to (A.2), (A.22) to get

$$\dot{V}(x)_{\substack{(A.2),\\(A.22)}} = \sum_{i=1}^{N} \left[-x_i^T G_i x_i - 2\rho x_i^T H_i B_i B_i^T H_i x_i + 2x_i^T H_i B_i \tilde{P}_i f_i(x) \right] \quad \forall x \in \mathcal{R}^n. \tag{A.24}$$

Now, complete the square involving the last two terms on the right hand side to get

$$\begin{aligned} \dot{V}(x)_{\substack{(A.2),\\(A.22)}} = & \sum_{i=1}^{N} \left[-x_i^T G_i x_i - \rho x_i^T H_i B_i B_i^T H_i x_i \right. \\ & - \rho x_i^T [H_i B_i - \rho^{-1} \tilde{P}_i f_i(x)]^T [H_i B_i - \rho^{-1} \tilde{P}_i f_i(x)] x_i \\ & \left. + \rho^{-1} f_i^T(x) \tilde{P}_i^T \tilde{P}_i f_i(x) \right] \\ & \quad \forall x \in \mathcal{R}^n. \end{aligned} \tag{A.25}$$

Employing inequality (A.3), and dropping negative terms on the right hand side, we have

$$\begin{aligned} \dot{V}(x)_{\substack{(A.2),\\(A.22)}} = & \sum_{i=1}^{N} \left[-x_i^T G_i x_i + \sum_{j=1}^{N} \rho^{-1} \|\tilde{P}_i\|^2 \eta_{ij} \|x_i\| \|x_j\| \right] \\ & \quad \forall x \in \mathcal{R}^n. \end{aligned} \tag{A.26}$$

or

$$\dot{V}(x)_{\substack{(A.2),\\(A.22)}} \leq -\mathbf{x}^T (\mathbf{G} - \rho^{-1} \mathbf{P}) \mathbf{x} \quad \forall x \in \mathcal{R}^n \tag{A.27}$$

where $\mathbf{G} = \mathrm{diag}\{\lambda_{\min}(G_1), \lambda_{\min}(G_2), \ldots, \lambda_{\min}(G_N)\}$ and $\mathbf{P} = \left[\|\tilde{P}_i\|^2 \eta_{ij} \right]$. In (A.27) we can choose ρ sufficiently large to make the matrix $(\mathbf{G} - \rho^{-1}\mathbf{P})$ positive definite, thereby making the right hand side less than zero. Therefore $V(x)$ is a Lyapunov function for the system (A.2), (A.22) (with sufficiently large ρ), and the system is stable.

We have thus shown that if interconnection disturbances enter within the range space of the controls, high gain decentralized feedback of the form (A.22) can stabilize the overall system. This is the basis by which we prove the stability of the adaptive algorithms in Chapters 3 and 5.

B
Adaptive Control

In this appendix we develop the theory of model-reference adaptive control while *ignoring* any disturbance or destabilizing effect of interconnections between subsystems. The purpose is to demonstrate the design technique without the added complications due to interconnections. The basis for this matrial can be found in the papers by Parks [9] and Monopoli [11].

We consider the isolated, single-input single-output system described by

$$\begin{aligned} \mathbf{S} : \dot{x} &= Ax + bu \\ y &= c^T x \end{aligned} \tag{B.1}$$

where $x(t) \in \mathcal{R}^n$, $u(t) \in \mathcal{R}$ and $y(t) \in \mathcal{R}$ are the state, input, and output of $\mathbf{S}$ respectively at time $t \in \mathcal{R}$, A is a constant $n \times n$ matrix, and b, and c are constant vectors of length n. For the sake of simplicity in the arguments that follow, we assume that the pair (A, b) is in controllable cannonical form, that is

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdot & 0 \\ 0 & 0 & 1 & \cdot\cdot & 0 \\ \cdots & \cdots & \cdots & \cdot & \cdot\cdot \\ -a_1 & -a_2 & -a_3 & \cdot & -a_n \end{pmatrix}, b = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ b_1 \end{pmatrix} \tag{B.2}$$
$$c^T = (\quad c_1 \quad c_2 \quad \ldots \quad c_n \;).$$

but that the $2n + 1$ constant coefficients $\{a_1, a_2, \ldots, a_n, b_1, c_1, c_2, \ldots, c_n\}$ are not known by the designer, hence the need for an adaptive control law.

The control objective is to either regulate the state, $x(t)$ of the system to zero, or to force $x(t)$ to track the state of a given reference model. The model for $\mathbf{S}$ is given by

$$\begin{aligned} \mathbf{M} : \dot{x}_m &= A_m x_m + b_m r \\ y_m &= c_m^T x_m . \end{aligned} \tag{B.3}$$

where $x_m(t) \in \mathcal{R}^n$ and $y(t) \in \mathcal{R}$ are the state and output of **M** respectively at time $t \in \mathcal{R}$, $r(t) \in \mathcal{R}$ is a piecewise continuous reference signal, A_m is a constant stable matrix, and b_m and c_m are constant n vectors. The reference model is given in the controller canonical form of (B.2) with the coefficients selected by the designer so that **M** has the stability characteristics we would like **S** to have.

There exists a constant vector $k^* \in \mathcal{R}^n$ and a constant scalar $k_0^* \in \mathcal{R}$ such that

$$A_m = A + bk^{*T}, \quad b_m = bk^*. \tag{B.4}$$

It is assumed that k_0^* is positive. Owing to the stability of **M**, there is a positive definite matrix H which solves the Liapunov equation

$$A_m^T H + HA_m = -G \tag{B.5}$$

for any given positive definite matrix G. To simplify our argument further, we will assume that the full state x of **S** is available for feedback, and that $c_m = c$. Now we use the control law

$$u = \theta^T \nu \tag{B.6}$$

where $\nu(t) \in \mathcal{R}^{n+1}$:

$$\nu = (e^T, r^T)^T, \tag{B.7}$$

$e = x - x_m$ is the tracking error and $\theta(t) \in \mathcal{R}^{n+1}$ is a vector of *estimates* of the constant vector $\theta^* = (k^{*T}, k_0^T)^T$. Next, we substitute (B.5) into (B.1), and subtract (B.3) to get a tracking error model

$$\begin{aligned} \mathbf{S}_e : \dot{e} &= A_m e + b\phi^T \nu \\ y &= c_m^T e. \end{aligned} \tag{B.8}$$

where $\phi = \theta - \theta^*$.

To derive an adaptive law for $\theta(t)$, we attempt to prove the stability of the system $\mathbf{S}_e$ using the second method of Liapunov. Starting with the candidate Liapunov function

$$V(e, \phi) = e^T He + \phi^T \Gamma^{-1} \phi \tag{B.9}$$

where Γ is any positive definite $(n+1) \times (n+1)$ matrix, we take the total time derivative of $V(e, \phi)$ along the solutions of $\mathbf{S}_e$ to get

$$\dot{V}(e,\phi)_{(B.8)} = -e^T G e + 2e^T H b \phi^T \nu + 2\phi^T \Gamma^{-1} \dot{\phi} \qquad \forall (t, e, \phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^{n+1}. \tag{B.10}$$

Note that $e^T H b$ is a scalar. Now, if we let the adaptive gains obey the differential equation

$$\dot{\phi} = \dot{\theta} = -\Gamma(e^T H b)\nu \tag{B.11}$$

then the second and third terms on the right hand side of (B.10) cancel and we have

$$\dot{V}(e,\phi)_{(B.8)} = -e^T G e \leq 0 \qquad \forall (t, e, \phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^{n+1} \tag{B.12}$$

thus $V(e, \phi)$ is non increasing, which implies that $e(t)$ and $\phi(t)$ are bounded for all $t > t_0$. The bounded solutions imply, in view of (B.8) and (B.11), that $\dot{e}(t)$ and $\dot{\phi}(t)$ are bounded, hence $\ddot{V}(e, \phi)$ is bounded and therefore $\dot{V}(e, \phi)$ is uniformly continuous on $\mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^{n+1}$. Furthermore, the function $V(t) = V[x(t), \phi(t)]$ is decreasing and is bounded from below. Hence,

$$\lim_{t \to \infty} V(t) = \inf_t V(t) = V_f \geq 0. \tag{B.13}$$

Denoting $\dot{V}(t) = \dot{V}[x(t), \phi(t)]_{(B.8)}$, we have

$$\lim_{t \to \infty} \int_{t_0}^{t} \dot{V}(\tau) d\tau = V_f - V_0 < \infty \tag{B.14}$$

where $V_0 = V(x_0)$. Since $\dot{V}(t)$ is uniformly continuous, the integrand of (B.14) must vanish as $t \to \infty$. Thus we have $\lim_{t \to \infty} \dot{V}(t) = 0$, therefore, in light of (B.12), $\lim_{t \to \infty} e(t) = 0$ for any given initial condition $(t_0, x_0, \phi_0) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n$.

We have shown that (B.11) is the stabilizing adaptation law for the variable gains θ. With this law, and under conditions where there is no external disturbance or interconnection to other systems, the state $x(t)$ of $\mathbf{S}$ will converge to the state $x_m(t)$ of the model $\mathbf{M}$ as $t \to \infty$, that is, the tracking error will converge to zero.

As an example, we consider adaptively controlling the unstable system

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{B.15}$$

so that is follows the reference model

$$A_m = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix},\ b_m = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{B.16}$$

Solving (B.5) with $G = I_2$, we get

$$H = \begin{pmatrix} 1.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}. \tag{B.17}$$

For the adaptive law, we choose $\Gamma = I_3$ (the 3×3 identity matrix), and use the reference signal

$$r(t) = \sin 20t + \sin 5t + \sin t. \tag{B.18}$$

The results of a simulation are plotted in Figure B 1, which shows tracking error and gains as a function of time. Note that tracking error goes to zero and gains tend toward the model-matching values of

$$k^* = \begin{pmatrix} -2 \\ -2 \end{pmatrix},\ k_0^* = 1. \tag{B.19}$$

The adaptive control law derived above is globally assymptotically stable for any unknown system **S** under the conditions stated. Unfortunately, however, the combined equations (B.8), (B.11) describe a *nonlinear* system, and, in this case, global assymptotic stability will not guarantee boundedness of solutions if the system is driven by an external disturbance signal. Since in practical applications, there is always some noise present, we must analyze the resulting behavior of the adaptively controlled system.

Consider the uniformly bounded, unknown disturbance signal $d(t)$ driving the system **S**

$$\mathbf{S} : \dot{x} = Ax + bu + d. \tag{B.20}$$

Using adaptive law (B.11) we have the error model

$$\mathbf{S}_e : \dot{e} = A_m e + b_m \phi^T \nu + d \tag{B.21}$$

Persuing the line of reasoning (B.9) through (B.12) we get

$$\begin{aligned} \dot{V}(e,\phi)_{\substack{(B.21),\\(B.11)}} &= -e^T Ge + 2e^T Hd \\ &\forall (t,e,\phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^{n+1}. \end{aligned} \tag{B.22}$$

from which we cannot conclude the convergence of tracking errors, nor even the boundedness of solutions $[e(t), \phi(t)]$.

Indeed, we see from simulation, that solutions may diverge in the presence of a bounded disturbance. Under the conditions of the example above, we have applied the disturbance vector $d(t) = (d_1, d_2)^T$, where $d_1 = 0$ and d_2 is uniformly distributed white noise on the interval $[-1.5, 1.5]$. Simulation results are shown in Figure B.2. The effect of the distrubance on the adaptive system is to continually perturb the tracking error away from zero. As a result, the adapted gains drift to larger and larger values, well beyond the model-matching values, with no guarantee of finite limits. In view of the adaptation law (B.11) we realize that the adaptive controller is not satisfied with the disturbance rejection the model would provide. Instead, ***adaptation will cease only when tracking error is identically zero.***

To make adaptive controllers robust with respect to bounded disturbances, the so-called σ-modification [18] is introduced into the adaptive law as follows

$$\dot{\theta} = -\Gamma(e^T Hb)\nu - \Gamma\sigma\theta. \tag{B.23}$$

As shown in the main part of this thesis, the σ-modification is crucial for robustness to interconnections between decentrally controlled subsystems. For now, we consider the uniformly bounded external disturbance $d(t)$ input to the isolated system $\mathbf{S}$ as before. The total time derivative of $V(e,\phi)$ with respect to (B.21) and (B.23) is now

$$\begin{aligned} \dot{V}(e,\phi)_{\substack{(B.21),\\(B.23)}} &= -\,e^T Ge + 2e^T Hd - 2\sigma\phi^T\phi - 2\sigma\phi^T\theta^* \\ &\forall (t,e,\phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^{n+1}. \end{aligned} \tag{B.24}$$

by completing squares, this becomes

$$\begin{aligned}\dot{V}(e,\phi)_{\substack{(B.21),\\(B.23)}} =& -\frac{1}{2}e^T G e - \frac{1}{2}(e - 2G^{-1}Hd)^T G(e - 2G^{-1}Hd) + 2d^T HG^{-1}Hd \\ & - \sigma\phi^T\phi - \sigma(\phi+\theta^*)^T(\phi+\theta^*) + \sigma\theta^{*T}\theta^*. \\ & \forall(t,e,\phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^{n+1}.\end{aligned} \tag{B.25}$$

After dropping the negative completed squares, and taking norms, we have

$$\begin{aligned}\dot{V}(e,\phi)_{\substack{(B.21),\\(B.23)}} \leq & -\frac{1}{2}\lambda_{\min}(G)||e||^2 + 2\frac{\lambda^2_{\max}(H)}{\lambda_{\min}(G)}\bar{d}^2 \\ & - \sigma||\phi||^2 + \sigma||\theta^*||^2 \\ & \forall(t,e,\phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^{n+1}.\end{aligned} \tag{B.26}$$

where $\bar{d} = \sup_t[d(t)]$, and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of the indicated matrices, respectively. Using the definition (B.9) we can write the above inequality more compactly as

$$\begin{aligned}\dot{V}(e,\phi)_{\substack{(B.21),\\(B.23)}} \leq & -\mu V + \eta \\ & \forall(t,e,\phi) \in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^{n+1}.\end{aligned} \tag{B.27}$$

where

$$\begin{aligned}\mu &= \min\{\frac{1}{2}\frac{\lambda_{\min}(G)}{\lambda_{\max}(H)}, \sigma\lambda_{\min}(\Gamma)\} \\ \eta &= 2\frac{\lambda^2_{\max}(H)}{\lambda_{\min}(G)}\bar{d}^2 + \sigma||\theta^*||^2\end{aligned} \tag{B.28}$$

Thus $V[e(t),\phi(t)]$ is monotonically decreasing function of time until $V < V_f$ where

$$V_f = \mu^{-1}\eta, \tag{B.29}$$

which implies that solutions to $\mathbf{S}_e$ are uniformly ultimately bounded, and converge to the residual set

$$\Omega_f = \{(e,\phi) : V(e,\phi) \leq V_f\} \tag{B.30}$$

Thus we see that adaptive laws using the σ-modification as shown will tolerate a bounded external disturbance. The tracking error will not in general converge to zero, but instead will reduce to below some residual bound, as determined by the set Ω_f, which will depend on the magnitude of the disturbance, the choice of reference model, the magnitude of θ^*, and the choice of design constants σ and Γ.

The effect of the σ-modification on the disturbed example system is shown in Figure B.3. Here we used the modified adaptive law (B.23) with $\sigma = 0.01$. Note that tracking errors converge to within a bounded, nonzero interval and that adapted gains also remain bounded. As shown in this thesis, there is a trade off between reducing tracking errors and having high (but finite) adapted gains.

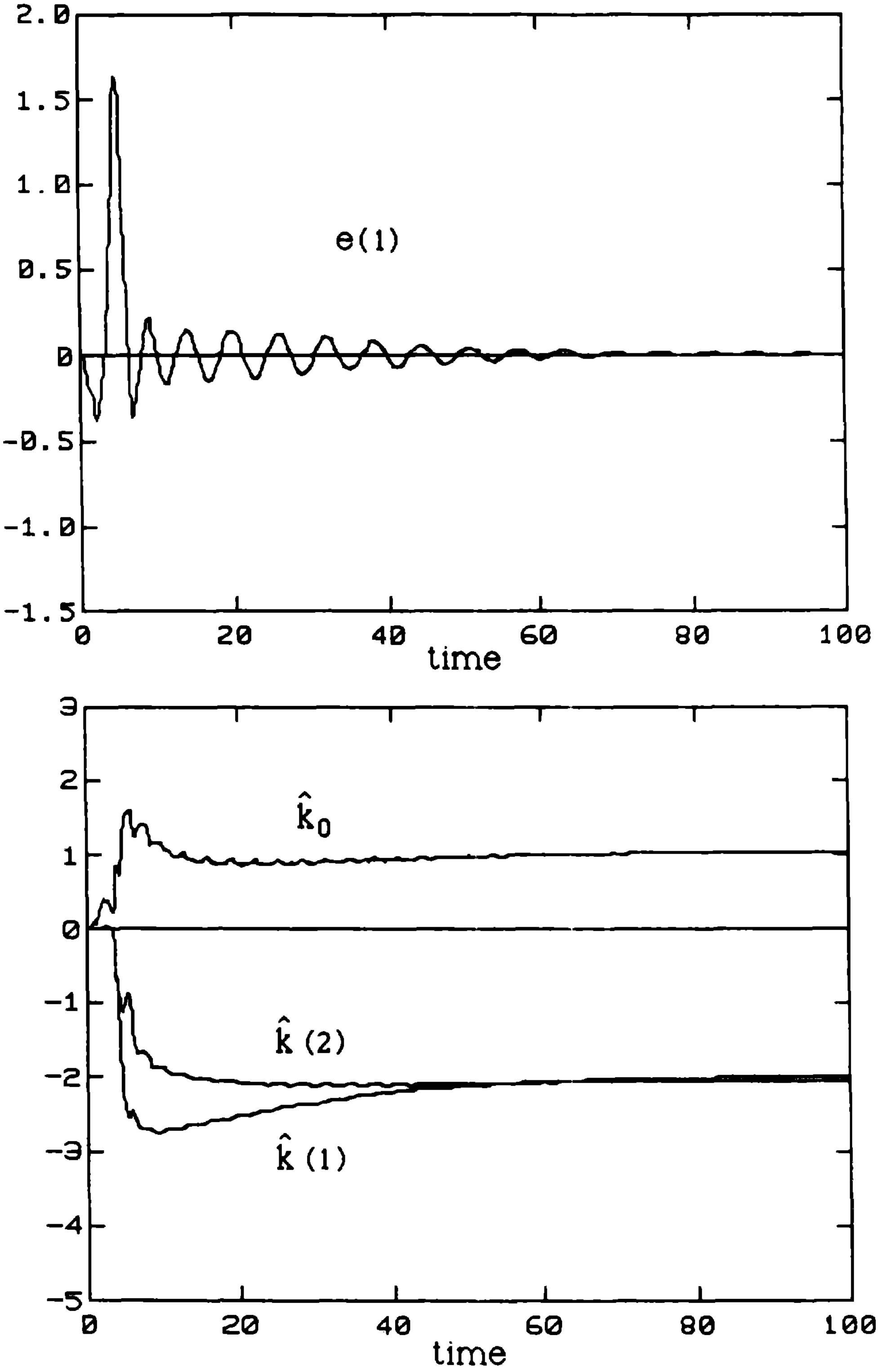

Figure B.1. Tracking error and gains for an isolated system.

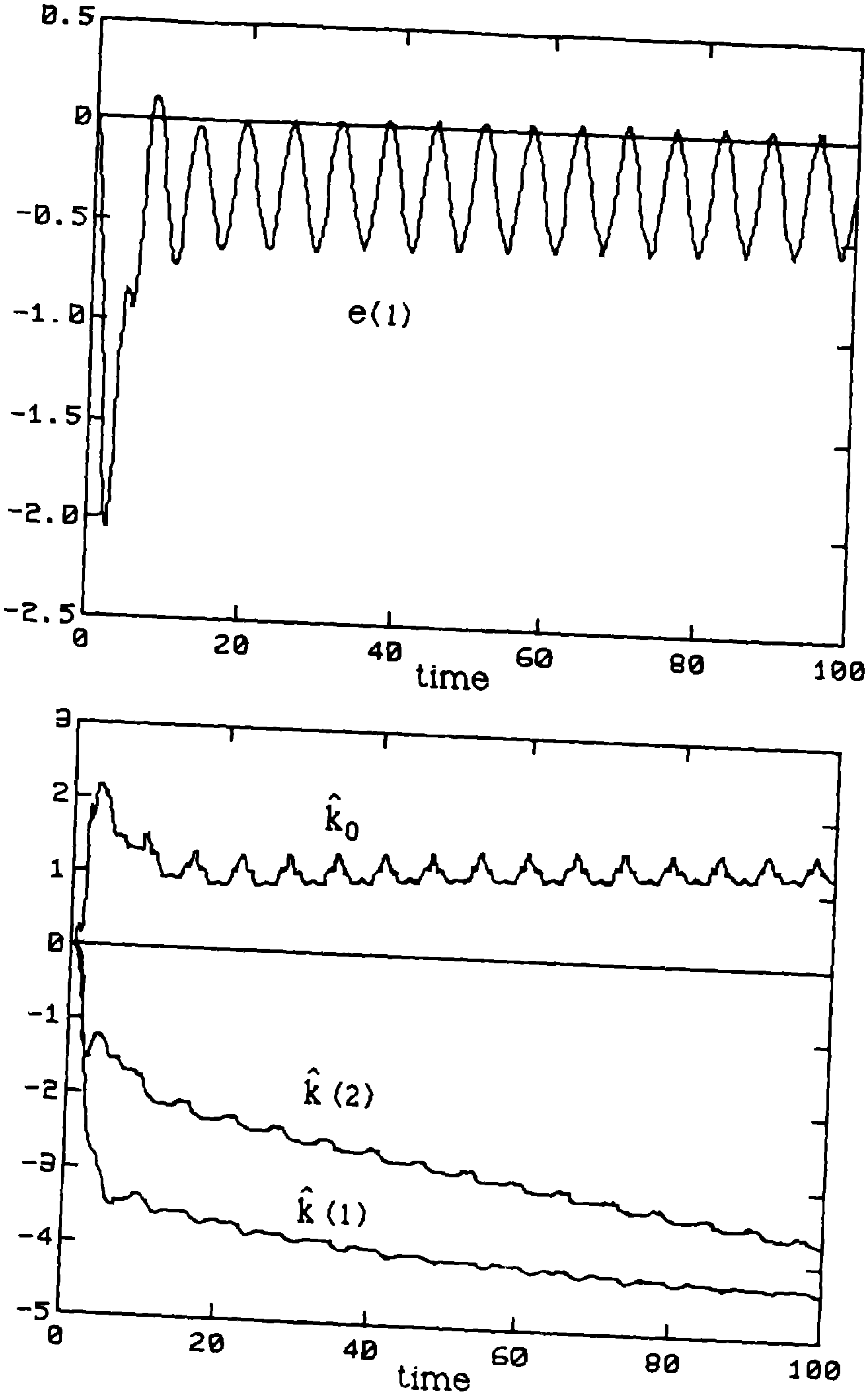

Figure B.2. Tracking error and gains with a bounded disturbance.

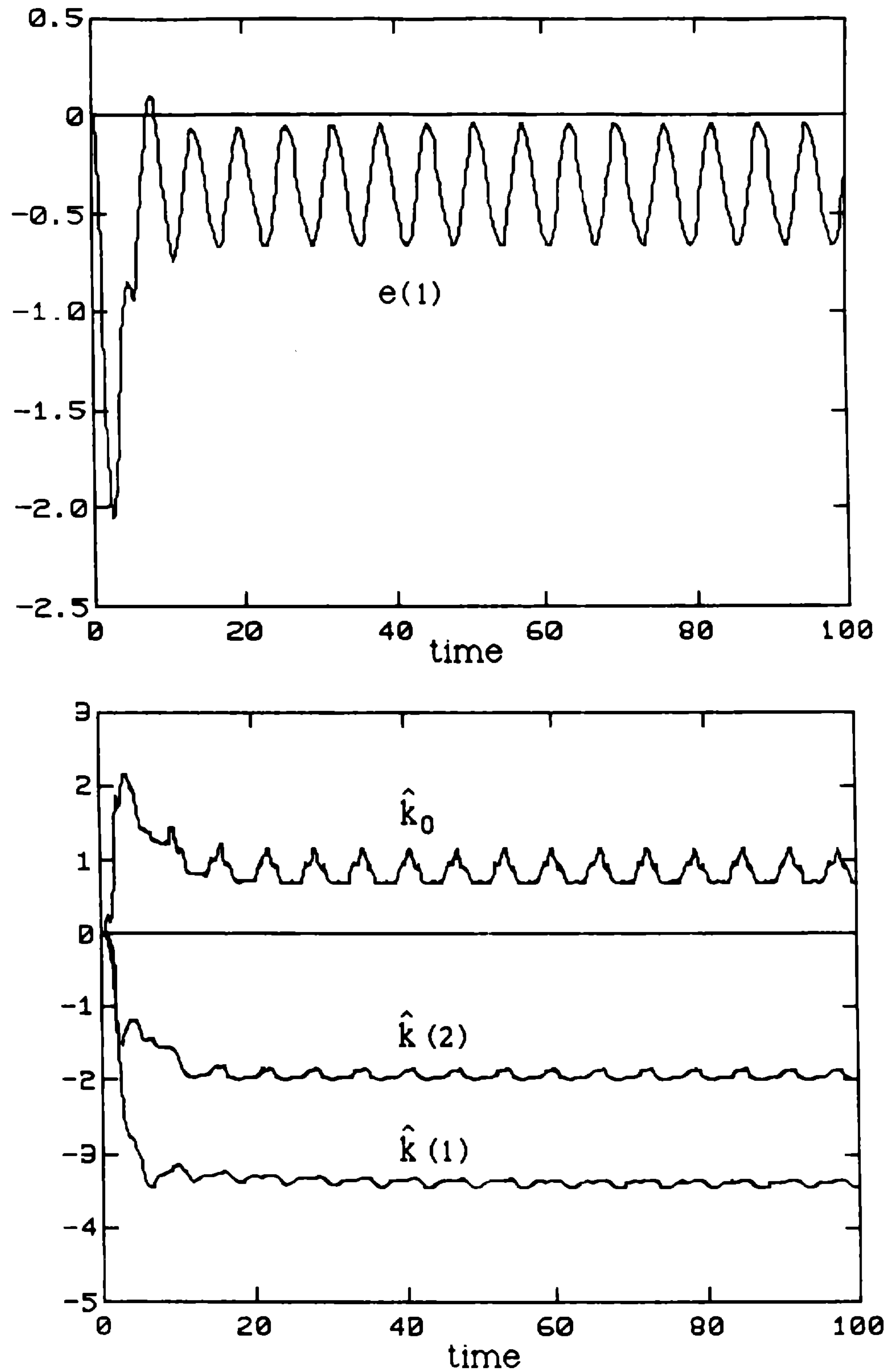

Figure B.3. Tracking error and gains with the σ-modification.